Energy Efficient Resource Allocation for Type-I HARQ Under the Rician Channel

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Abstract

This paper addresses the per-link power and bandwidth allocation problem with the objective of maximizing energy efficiency (EE) related metrics under a per-link minimum goodput constraint when only statistical channel state information is available. We consider a parallel (i.e., without multiuser interference) Rician channel model, which encompasses both Rayleigh and additive white Gaussian noise channels as special cases. We also consider Type-I hybrid automatic repeat request with practical modulation and coding schemes. The addressed problems are the maximization of the sum of the user's EE, the maximization of the EE of the user with the lowest EE and the maximization of the EE of the network. We derive the optimal solutions of these problems in closed form using fractional programming and a convex optimization framework. We show that substantial gains can be achieved by taking into account the line of sight between the transmitter and the receiver instead of only considering the average channel power.

Index Terms

HARQ, resource allocation, energy efficiency, fractional programming, convex optimization, D2D communications.

I. Introduction

Modern wireless communications often take place in a multiuser context. The performance of multiuser systems is led by so-called resource allocation (RA), which requires some knowledge

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about the links' channels. In this work, we consider that only statistical channel state information (CSI) is available to perform RA. This assumption is realistic, for instance, in ad hoc networks [3], or in device to device (D2D) communications, which are of central importance within 5G networks [4], [5]. Indeed, in these types of networks, RA may be performed in an assisted fashion, i.e., there is a node called the resource manager (RM) performing RA [6]. As a consequence, the RM, which collects all the links' CSI, has only access to outdated CSI. Instead of performing RA using unreliable outdated CSI, we consider that the RM uses statistical CSI to perform RA [3]. Hence, the underlying statistical channel model is crucial. In this paper, we especially focus on the Rician fading channel, which is known to accurately represent the realistic statistical behavior of wireless channel when there exists a line of sight (LoS) between the transmitter and the receiver [7], [8]. This channel model encompasses both the Rayleigh and the additive white Gaussian noise (AWGN) channels as special cases. The Rician channel model receives more attention today in the literature due to its accuracy of modeling the channel in the context of millimeter wave communications [7], [9], which is a promising technology for 5G communications [4].

Since we have access only to statistical CSI, we consider the use of the hybrid automatic repeat request (HARQ) mechanism to increase the reliability of the communications. This mechanism is a combination of automatic repeat request (ARQ) and forward error correction (FEC) allowing the improvement of the transmission capability. Actually, FEC provides a correction capability while ARQ allows the system to take advantage of the time varying nature of the wireless channel. The HARQ mechanism is used today in a multiuser context, for instance in 4G long term evolution (LTE) [10]. There exist different types of HARQ, differing in the way the packets are sent and decoded [11]. In this paper, we find the optimal closed form solutions of the RA problems for Type-I HARQ, which is the simplest HARQ protocol since the packets received in error are discarded. We also numerically evaluate the performance of our proposed solutions to Type-II HARQ, which keeps packets received in error to help the decoding step.

RA is performed by optimizing a criterion subject to quality of service (QoS) constraints. Two well established criteria are the minimization of the total transmit power [12] and the maximization of the data rate [13]. Although it is clear that maximizing the data rate leads to high energy consumption and is in general not energy efficient, it is less obvious that minimizing the transmit power may not be efficient from an energy viewpoint. these reasons, another metric called energy efficiency (EE) has appeared in the literature and has recently gained much interest [14], [15]. This metric, measured in bits/joule, is defined as the ratio between the goodput, i.e., the number of information bits that can be transmitted without error per unit of time, and the power consumption to transmit these bits. Hence, the EE is of interest since it captures a trade off between the goodput and the energy consumption. In this paper, we are interested in maximizing EE related metrics for HARQ-based systems when only statistical CSI is available. In details, we focus on maximizing the sum of the users' EE (SEE), the EE of the network, called global EE (GEE) and the EE of the link with the lowest EE called minimum EE (MEE). In the following, we introduce some notations extensively used throughout this paper, along with insights regarding the metrics relevancy.

- The SEE maximization, referred to as the maximum SEE (MSEE), is user-centric and it is flexible since one can plug weights in the sum to represent the links' priority (although the choice of the weights is not discussed in this paper).
- The GEE maximization, referred to as the maximum GEE (MGEE) is of interest as long as maximizing the network EE is at stake.
- The MEE maximization, referred to as the maximum MEE (MMEE), is interesting when achieving high fairness among the links' EE is the objective, i.e., when each links' EE is crucial.

Our objective is to allocate to each link a transmit power and a bandwidth proportion. We consider a parallel channel model, i.e., there is no multiuser interference.

A. Related Works

First, let us review the works studying the EE of HARQ in the single user context [16]–[29]. In [16]–[27], the authors consider statistical CSI at the transmitter. In contrast, imperfect CSI is assumed in [28] while perfect CSI is assumed to be available in [29]. Notice that, in [29], the authors do not explicitly consider the HARQ mechanism, but the considered metric is valid for Type-I HARQ. These works mainly address power and/or rate optimization between transmissions related to the same information packet, typically using convex optimization.

Second, we focus on the works dealing with RA with EE related criteria in a multiuser context when considering perfect CSI at the transmitter side. In this category, a lot of works consider the use of capacity achieving codes [30]–[38] whereas practical modulation and coding schemes (MCS) are considered in [39]. Among those works, [30]–[36], [39] does not consider HARQ whereas this mechanism is taken into account in [37], [38]. In details, when capacity achieving codes are considered with no HARQ, the MSEE problem is solved in [30] while the

MMEE problem is solved in [31]. In [32], several heuristics are derived for the MSEE and MMEE problems. The multi-cell problem is addressed in [33], [34]. In [33], the MSEE and MGEE problems are solved while, in [36], the MEE maximization problem is addressed. In [35], centralized and decentralized algorithms are proposed for the MGEE problem. In [34], a distributed algorithm is proposed to solve the MMEE problem. When capacity achieving codes are considered with HARQ and perfect CSI [37], [38], the GEE is optimized in [37] while several RA schemes are investigated in [38]. When practical MCS along with perfect CSI are considered without HARQ, the MGEE problem for the LTE downlink is addressed in [39].

Third, we review the works addressing the RA problem with EE related objective functions when statistical CSI is available. This problem is addressed considering capacity achieving codes with no HARQ and the Rayleigh channel in [40], [41]. Practical MCS with HARQ are considered in [42], [43], which consider the Rayleigh channel. In [42], the authors maximize the harmonic mean of the users' EE in a relay assisted networks when Type-I HARQ is considered. In [43], the MSEE problem is solved for Type-II HARQ.

Finally, Table I summarizes the existing works concerning the RA problem with EE related metrics for HARQ when considering practical MCS. We see that no work addresses the RA problem with the objective of maximizing EE related metrics under the Rician channel when statistical CSI is available.

TABLE I

EXISTING EE-BASED RA SOLUTIONS FOR HARQ WITH PRACTICAL MCS AND STATISTICAL CSI IN THE MULTIUSER CONTEXT.

	Rayleigh	Rice
Type-I	[42], [43]	None
Type-II	[43]	None

B. Contributions and Paper Organization

The contributions of this paper are the following ones:

- We optimally solve the MSEE, MGEE and MMEE problems for Type-I HARQ under the Rician channel. Actually, we manage to transform these problems which have no interesting properties like convexity into convex problems. Our main technical contribution is to provide the analytical optimal solutions of these convex problems using the Karush-Kuhn-Tucker (KKT) conditions.

- We analyze the results of the proposed criteria through numerical simulations, and point out that substantial EE gains can be achieved by taking into account the Rician channel instead of the conventional Rayleigh channel. In other words, we exhibit the importance of taking into account the existence of a LoS during RA instead of only considering the channel average power.
- We numerically study RA solutions for Type-II HARQ under the Rician channel. Actually, from Table I, we see that RA for Type-II HARQ under the Rayleigh channel is done in [43], while in this paper, we perform RA for Type-I HARQ under the Rician channel. We compare RA from [43] and the one from this paper when applied on Type-II HARQ under the Rician channel.

The rest of the paper is organized as follows. In Section II, we present the system model, our main assumptions and the RA problems. In Section III, we explain the solution methodology used to solve these problems. In Section IV, we optimally solve the MSEE problem. In Section V, we optimally solve the MGEE problem. In Section VI, we optimally solve the MMEE problem. In Section VII, we study the results of the proposed criteria and the solutions to extend our work to Type-II HARQ through numerical simulations. Finally, in Section VIII, we draw concluding remarks. For the ease of readability, all the proofs are provided in the Appendices.

II. System Model, Assumptions and Problem Formulation

A. Channel Model and HARQ Mechanism

Let us consider a network with a total bandwidth B divided into N_c subcarriers, which are shared between L active links using the orthogonal frequency division multiple access (OFDMA) as the multiple access technology. We assume a parallel channel model, i.e., there is no multiuser interference. Notice that our derivations extend straightforwardly to any multiple access multicarrier scheme and to single-carrier frequency division multiplexing as long as there is no interference between users. We suppose that the RM centralizes the statistical CSI of the links to perform RA. We consider for each link a multipath channel, which is constant within one orthogonal frequency division multiplexing (OFDM) symbol and varies independently from OFDM symbol to OFDM symbol. Notice that although this assumption is ideal, all the solutions derived in this paper are applicable to channel with longer channel coherence time as long as an analytical error probability approximation fulfilling the mild assumptions detailed in Section II-D is available.

Similarly to [44], the received signal on link ℓ on the nth subcarrier at OFDMA symbol i can be written as

$$Y_\ell(i,n) = H_\ell(i,n)X_\ell(i,n) + Z_\ell(i,n), \tag{1}$$

with $H_\ell(i,n) \sim \mathcal{CN}(a_\ell, \zeta_\ell^2)$ where $\mathcal{N}(a_\ell, \zeta_\ell^2)$ stands for the complex Gaussian distribution with mean a_ℓ and variance ζ_ℓ^2, $X_\ell(i,n)$ is the transmitted symbol on the nth subcarrier of the ith OFDMA symbol and $Z_\ell(i,n) \sim \mathcal{CN}(0, N_0 B/N_c)$, with N_0 the noise power spectral density. We assume that $H_\ell(i,n)$ is known at the receiver of the ℓth link. We can define the average gain-to-noise ratio (GNR) G_ℓ and Rician K factor K_ℓ of the ℓth link as

$$G_\ell := \frac{\mathbb{E}[|H_\ell(i,n)|^2]}{N_0} = \frac{\Omega_\ell}{N_0}, \tag{2}$$

$$K_\ell := \frac{|a_\ell|^2}{\zeta_\ell^2}, \tag{3}$$

with $\Omega_\ell := |a_\ell|^2 + \zeta_\ell^2$, where $:=$ means by definition. Notice that $K_\ell = 0$ corresponds to the Rayleigh channel while $K_\ell \to +\infty$ corresponds to the AWGN channel. It is assumed that the RM only knows the average GNR and the Rician K factor of each link to perform RA. Moreover, we suppose that well-designed time and frequency interleavers are used such that each modulated symbol experiments experiences an independent channel realization, i.e., the channel can be seen as fast fading.

We assume that, at the medium access (MAC) layer, each link uses a Type-I HARQ scheme. The information bits are grouped into packets of $\mathcal{L}_\ell$ bits, which are encoded by an FEC with rate R_ℓ to obtain the MAC packets. A MAC packet is sent on the channel at most $\mathcal{T}$ times. At the received side, after decoding the mth received packet, the information bits are checked using a cyclic redundancy check (CRC) which is assumed to be error free. An acknowledgement (ACK) is sent if the information bits are correctly decoded, while otherwise a negative ACK (NACK) is sent. Since Type-I HARQ is considered, the MAC packets received in error are discarded.

B. Energy Consumption Model

We suppose that a quadrature amplitude modulation (QAM) modulation with m_ℓ bits per symbol is used on link ℓ. Let n_ℓ and $\gamma_\ell := n_\ell/N_c$ be the number of subcarriers and the bandwidth proportion allocated to the ℓth link, respectively. Because the channel coefficients on each subcarrier are identically distributed, the same power is used on all the subcarriers, and we allocate bandwidth proportion instead of specific subcarriers. We then define $P_\ell := \mathbb{E}[|X_\ell(j,n)|^2]$ as the power allocated per subcarrier to the ℓth link[1].

The total energy consumed to transmit and receive one packet is the sum of the transmission energy and the circuitry consumption of both the transmitter and the receiver. The power used by the ℓth link to transmit and receive one OFDMA symbol is

$$P_{T,\ell} := N_c \gamma_\ell P_\ell \kappa_\ell^{-1} + P_{ctx,\ell} + P_{crx,\ell}, \tag{4}$$

with $\kappa_\ell \leq 1$ the efficiency of the power amplifier and $P_{ctx,\ell}$ (resp. $P_{crx,k}$) the circuitry power consumption of the transmitter (resp. the receiver).

C. *Energy Efficiency*

The EE $\mathcal{E}_\ell$ of link ℓ is the ratio between its goodput η_ℓ and its power consumption:

$$\mathcal{E}_\ell := \frac{\eta_\ell}{P_{T,\ell}} \quad \frac{[\text{bits/s}]}{[\text{W}]}. \tag{5}$$

Similarly, the GEE is the ratio of the sum of the users' goodput and the sum of their power consumption, which writes:

$$\mathcal{G} := \frac{\sum_{\ell=1}^{L} \eta_\ell}{\sum_{\ell=1}^{L} P_{T,\ell}}. \tag{6}$$

For Type-I HARQ and when the channel has no correlation between the HARQ rounds, the goodput is given by [11]:

$$\eta_\ell = B \alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell E_\ell)), \tag{7}$$

where $q_\ell(x)$ is the packet error rate (PER) on the ℓth link for signal-to-noise ratio (SNR) x, $E_\ell := N_c P_\ell / B$ and $\alpha_\ell := m_\ell R_\ell$. Notice that because Type-I HARQ is considered and since the channel is uncorrelated between the HARQ round, (7) is independent of $\mathcal{T}$. By plugging (7) and (4) into (5) and (6), we obtain the following expressions of the EE and GEE:

$$\mathcal{E}_\ell(E_\ell, \gamma_\ell) = \frac{\alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell E_\ell))}{\kappa_\ell^{-1} \gamma_\ell E_\ell + C_\ell}, \quad \forall \ell, \tag{8}$$

$$\mathcal{G}(\mathbf{E}, \boldsymbol{\gamma}) = \frac{\sum_{\ell=1}^{L} \alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell E_\ell))}{\sum_{\ell=1}^{L} (\kappa_\ell^{-1} \gamma_\ell E_\ell + C_\ell)}, \tag{9}$$

where $C_\ell := (P_{ctx,\ell} + P_{crx,\ell})/B$, and $\mathbf{E} := [E_1, \ldots, E_L]$, $\boldsymbol{\gamma} := [\gamma_1, \ldots, \gamma_L]$ are the optimization variables, i.e., the resource that have to be allocated to the links.

[1]Notice that a signal is typically in Volts and thus expectation of its signal square (P_ℓ) might be in (Volts)^2. However, a power in Watt is proportional to (Volts)^2 up to an impedance and thus, for simplicity, we assume that P_ℓ is in Watt.

D. Assumptions on the Packet Error Rate

For future derivations, we make now three assumptions on the PER $q_\ell(x)$.

Assumption 1. *$q_\ell(x)$ is strictly convex, i.e., for all x in $\mathbb{R}^{+*}$, $q''_\ell(x) > 0$ with $q''_\ell(x)$ the second order derivative of $q_\ell(x)$.*

Assumption 2. *$q_\ell(x)$ is a strictly decreasing function on $\mathbb{R}^{+*}$, i.e., for all x in $\mathbb{R}^{+*}$, $q'_\ell(x) < 0$ with $q'_\ell(x)$ the first order derivative of $q_\ell(x)$.*

Assumption 3. $\lim_{x\to\infty} q_\ell(x) = 0$ *and* $\lim_{x\to\infty} q'_\ell(x) = 0$.

In Section VII devoted to simulations, we use the PER approximation provided in [44] for the Rician fast fading channel, which is given by:

$$q_\ell(x) = \left(c_\ell \sum_{i=1}^{4} \Omega_i \frac{e^{-\frac{(a_\ell)^2 x\theta_i d_\ell}{1+2\sigma_{h,\ell}^2 x\theta_i d_\ell}}}{1 + 2\sigma_{h,\ell}^2 x\theta_i d_\ell} \right)^{\zeta_\ell} e^{b_\ell}, \tag{10}$$

where θ_i and Ω_i are fitting coefficients related to approximation of Q-function, where ζ_ℓ and b_ℓ are fitting parameters depending on the considered MCS and on the Rician K factor which are provided in [44, Table I] whereas c_ℓ and d_ℓ are modulation dependent parameters available in [45, Table 6.1]. One can check that Assumptions 1-3 hold for (10).

E. Considered Constraints

In this paper, we consider a QoS constraint by imposing a target minimum goodput constraint per link, denoted by $\eta_\ell^{(t)}$, which can be expressed, for link ℓ:

$$B\alpha_\ell\gamma_\ell(1 - q_\ell(G_\ell E_\ell)) \geq \eta_\ell^{(t)}. \tag{11}$$

In addition, from the definition of the bandwidth parameter, the following inequality has to hold:

$$\sum_{\ell=1}^{L} \gamma_\ell \leq 1. \tag{12}$$

F. Problem Formulation

We address the MSEE, MGEE and MMEE problems, which write respectively as follows:

$$\textbf{P1:} \quad \max_{\mathbf{E},\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L} \frac{\alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell E_\ell))}{\kappa_\ell^{-1} \gamma_\ell E_\ell + C_\ell}, \qquad \text{s.t. } (11), (12),$$

$$\textbf{P2:} \quad \max_{\mathbf{E},\boldsymbol{\gamma}} \quad \frac{\sum_{\ell=1}^{L} \alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell E_\ell))}{\sum_{\ell=1}^{L} (\gamma_\ell E_\ell \kappa_\ell^{-1} + D_\ell)}, \qquad \text{s.t. } (11), (12),$$

$$\textbf{P3:} \quad \max_{\mathbf{E},\boldsymbol{\gamma}} \quad \min_{\ell \in \{1,\ldots,L\}} \frac{\alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell E_\ell))}{\kappa_\ell^{-1} \gamma_\ell E_\ell + C_\ell}, \qquad \text{s.t. } (11), (12).$$

In the rest of this paper, we address the optimal solution of Problems 1-3.

III. Solution Procedure

Problems P1-P3 have no special properties like convexity and thus they cannot be directly solved with affordable complexity. To overcome this issue, we propose the following change of variables, enabling us to apply convex optimization tools to solve them: $(\boldsymbol{\gamma}, \mathbf{E}) \mapsto (\boldsymbol{\gamma}, \mathbf{Q})$, with $\mathbf{Q} := [Q_1, \ldots, Q_L]$, and

$$Q_\ell := \gamma_\ell E_\ell, \quad \forall \ell. \tag{13}$$

Using (13), constraints (11) can be rewritten as

$$\eta_\ell^{(0)} \leq \alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell Q_\ell / \gamma_\ell)), \quad \forall \ell, \tag{14}$$

with $\eta_\ell^{(0)} := \eta_\ell^{(t)} / B$. Then, problems P1-P3 can be rewritten equivalently as follows:

$$\textbf{P1':} \quad \max_{\mathbf{Q},\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L} \frac{\alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell Q_\ell / \gamma_\ell))}{\kappa_\ell^{-1} Q_\ell + C_\ell}, \qquad \text{s.t. } (14), (12),$$

$$\textbf{P2':} \quad \max_{\mathbf{Q},\boldsymbol{\gamma}} \quad \frac{\sum_{\ell=1}^{L} \alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell Q_\ell / \gamma_\ell))}{\sum_{\ell=1}^{L} (\kappa_\ell^{-1} Q_\ell + C_\ell)}, \qquad \text{s.t. } (14), (12),$$

$$\textbf{P3':} \quad \max_{\mathbf{Q},\boldsymbol{\gamma}} \quad \min_{\ell \in \{1,\ldots,L\}} \frac{\alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell Q_\ell / \gamma_\ell))}{\kappa_\ell^{-1} Q_\ell + C_\ell}, \qquad \text{s.t. } (14), (12).$$

We characterize P1'-P3' in the following lemma, whose proof is provided in Appendix A.

Lemma 1. *The numerators of the objective functions of Problems P1' to P3' are jointly concave in* $(\boldsymbol{\gamma}, \mathbf{Q})$*, their denominators are jointly convex in* $(\boldsymbol{\gamma}, \mathbf{Q})$ *and the feasible set defined by* (14) *and* (12) *is jointly convex in* $(\boldsymbol{\gamma}, \mathbf{Q})$.

According to Lemma 1, we know that P1'-P3' (and thus P1-P3 since they are equivalent) can be optimally solved iteratively: P1' can be solved using the Jong's algorithm [46], P2' using the Dinkelbach's algorithm [47] and P3' using the generalized Dinkelbach's algorithm [48]. At each iteration, these three algorithms require solving a convex optimization problem. The major technical contribution of this paper is to provide the optimal solution of these convex optimization problems using the so-called KKT conditions [49].

For the three problems, our solution is organized as follows: we first express the optimal solution as a function of a single parameter, and second we find the optimal value of this parameter.

IV. MSEE Problem (P1)

We apply the Jong's algorithm [46], which has been used in the RA context with EE related metrics for instance in [30], [50], [51]. At iteration i, we have to solve the following problem:

$$\textbf{P1.1:}\quad \max_{\mathbf{Q};\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L} u_\ell^{(i)}\alpha_\ell\gamma_\ell(1-q_\ell(G_\ell Q_\ell/\gamma_\ell)) - u_\ell^{(i)}\beta_\ell^{(i)}(\kappa_\ell^{-1}Q_\ell + C_\ell), \qquad \text{s.t. } (14), (12),$$

where, $\forall \ell$, $u_\ell^{(i)} > 0$ and $\beta_\ell^{(i)} > 0$ depend on the optimal solution at iteration $(i-1)$.

P1.1 is the maximization of a concave function over a convex set. Hence, the KKT conditions are necessary and sufficient to find its global optimal solution [49]. Defining $\boldsymbol{\delta} := [\delta_1, \ldots, \delta_L]$ and λ as the Lagrangian multipliers associated with constraints (14) and (12), respectively, the KKT conditions of P1.1 write:

$$\alpha_\ell G_\ell q_\ell'\left(G_\ell Q_\ell/\gamma_\ell\right)(u_\ell^{(i)} + \delta_\ell) + u_\ell^{(i)}\beta_\ell^{(i)}\kappa_\ell^{-1} = 0, \quad \forall \ell, \tag{15}$$

$$\alpha_\ell h_\ell\left(G_\ell Q_\ell/\gamma_\ell\right)(u_\ell^{(i)} + \delta_\ell) + \lambda = 0, \quad \forall \ell, \tag{16}$$

with $h_\ell(x) := -1 + q_\ell(x) - x q_\ell'(x)$. In addition, the following complementary slackness conditions hold at the optimum:

$$\delta_\ell\left(\eta_\ell^{(0)} - \alpha_\ell\gamma_\ell\left(1 - q_\ell\left(G_\ell Q_\ell/\gamma_\ell\right)\right)\right) = 0, \quad \forall \ell, \tag{17}$$

$$\lambda\left(\sum_{\ell=1}^{L}\gamma_\ell - 1\right) = 0. \tag{18}$$

To solve the optimality conditions (15)-(18), in a first time we consider the value of λ as fixed and we find the optimal solution as a function of this multiplier. In a second time, we search for the optimal value of this multiplier.

A. Solution for Fixed λ

From (15), we obtain the following L relations:

$$u_\ell^{(i)} + \delta_\ell = \frac{-u_\ell^{(i)}\beta_\ell^{(i)}\kappa_\ell^{-1}}{\alpha_\ell G_\ell q_\ell'(x_\ell^*(\lambda))}, \quad \forall \ell, \tag{19}$$

with, $\forall \ell$, $x_\ell^*(\lambda) := G_\ell Q_\ell^*(\lambda)/\gamma_\ell^*(\lambda)$, where $Q_\ell^*(\lambda)$ (resp. $\gamma_\ell^*(\lambda)$) is the optimal Q_ℓ (resp. γ_ℓ) for given λ. Then, by plugging (19) into (16), we get

$$g_\ell(x_\ell^*(\lambda)) = \frac{\lambda}{u_\ell^{(i)}\beta_\ell^{(i)}\kappa_\ell^{-1}}, \quad \forall \ell, \tag{20}$$

with $g_\ell(x) := h_\ell(x)/(G_\ell q_\ell'(x))$. By computing the derivative of $g_\ell(x)$, one can prove that it is strictly increasing, allowing us to obtain $x_\ell^*(\lambda)$ using (20) as:

$$x_\ell^*(\lambda) = g_\ell^{-1}\left(\frac{\lambda}{u_\ell^{(i)}\beta_\ell^{(i)}\kappa_\ell^{-1}}\right), \quad \forall \ell. \tag{21}$$

We can then plug this optimal value into P1.1, which can be rewritten as:

$$\textbf{P1.2:} \quad \max_{\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L}\mathcal{K}_\ell(\lambda)\gamma_\ell, \quad \text{s.t.} \quad \gamma_\ell \geq \gamma_{\ell,\min}(\lambda), \forall \ell, \quad \text{and (12)},$$

with, $\forall \ell$, $\mathcal{K}_\ell(\lambda) := \alpha_\ell u_\ell^{(i)}(1 - q_\ell(x_\ell^*(\lambda))) - u_\ell^{(i)}\beta_\ell^{(i)}\kappa_\ell^{-1}x_\ell^*(\lambda)G_\ell^{-1}$ and $\gamma_{\ell,\min}(\lambda) := \eta_\ell^{(0)}/(\alpha_\ell(1 - q_\ell(x_\ell^*(\lambda))))$. P1.2 is a linear program depending only on the optimization variables $\boldsymbol{\gamma}$. In addition, since there is only one coupling constraint (12), its optimal solution is obtained straightforwardly according to the following two possible cases.

1) If, $\forall \ell$, $\mathcal{K}_\ell(\lambda) < 0$: $\forall \ell$, $\gamma_\ell^*(\lambda)$, the optimal value of $\gamma_\ell(\lambda)$, is given by $\gamma_\ell^*(\lambda) = \gamma_{\ell,\min}(\lambda)$.
2) If, $\exists \ell$, such that $\mathcal{K}_\ell(\lambda) \geq 0$: let $\ell_{M,\mathcal{K}}$ be such that, $\forall \ell$, $\mathcal{K}_{\ell_{M,\mathcal{K}}}(\lambda) \geq \mathcal{K}_\ell(\lambda)$. Then, $\forall \ell \neq \ell_{M,\mathcal{K}}$, $\gamma_\ell^*(\lambda) = \gamma_{\ell,\min}(\lambda)$ and $\gamma_{\ell_{M,\mathcal{K}}}^*(\lambda) = 1 - \sum_{\ell \neq \ell_{M,\mathcal{K}}}\gamma_{\ell,\min}(\lambda)$.

B. Search for the Optimal λ

To find λ^*, the optimal value of λ, we need to identify two possibilities: either there exists ℓ such that $\delta_\ell = 0$, or $\forall \ell$, $\delta_\ell > 0$. In the following, we discuss these two possible cases.

Case 1: $\exists \ell$ *such that* $\delta_\ell = 0$. In the following lemma whose proof is provided in Appendix B, we exhibit λ^*.

Lemma 2. *If there is at least one link* ℓ_1 *with* $\delta_{\ell_1} = 0$*, then we have*

$$\lambda^* = -\arg\min_{\ell} \{\alpha_\ell u_\ell h_\ell(x^*_{\ell,\delta_\ell=0})\}, \tag{22}$$

with $x^*_{\ell,\delta_\ell=0} := q'^{-1}_\ell(-\beta^{(i)}_\ell \kappa^{-1}_\ell / (\alpha_\ell G_\ell))$.

Thanks to Lemma 2, we can optimally solve P1' solving P1.2 with low complexity. Moreover, this lemma enables us to check whether $\exists \ell$ such that $\delta_\ell = 0$ constraint by computing λ^* and plugging it into P1.2. It the resulting problem is feasible, then we know that $\exists \ell$ such that $\delta_\ell = 0$.

Case 2: $\forall \ell, \delta_\ell > 0$. In this case, $\gamma^*_\ell(\lambda)$ can be obtained more easily using (17), which gives us

$$\gamma^*_\ell(\lambda) = \frac{\eta^{(0)}_\ell}{1 - q_\ell(x^*_\ell(\lambda))}, \quad \forall \ell, \tag{23}$$

where $x^*_\ell(\lambda)$ is given by (21). Since $g_\ell(x)$ is strictly increasing, $x^*_\ell(\lambda)$ is also increasing, implying that $\gamma^*_\ell(\lambda)$ is strictly decreasing due to (23). To find λ^*, we use the complementary slackness condition (18). To this end, we define the following function representing the sum of the optimal bandwidth parameters:

$$\Gamma(\lambda) := \sum_{\ell=1}^{L} \gamma^*_\ell(\lambda). \tag{24}$$

Since $\gamma^*_\ell(\lambda)$ is strictly decreasing, there are two possibilities: either $\Gamma(0) \leq 1$ and in this case $\lambda^* = 0$, or λ^* is such that $\Gamma(\lambda^*) = 1$. Hence, λ^* can be found by a one dimensional linesearch method.

C. Optimal Solution of MSEE (P1)

Gathering all the pieces together, the optimal solution of P1 is depicted in Algorithm 1, for which we define $\boldsymbol{\psi}(\boldsymbol{\beta}^{(i)}, \mathbf{u}^{(i)}, \boldsymbol{\gamma}, \mathbf{Q}) := [\psi_1(\beta^{(i)}_1, u^{(i)}_1, \gamma_1, Q_1), \ldots, \psi_{2L}(\beta^{(i)}_L, u^{(i)}_L, \gamma_L, Q_L)]$, with $\boldsymbol{\beta}^{(i)} := [\beta^{(i)}_1, \ldots, \beta^{(i)}_L]$ and $\mathbf{u}^{(i)} := [u^{(i)}_1, \ldots, u^{(i)}_L]$ and, for $\ell = 1, \ldots, L$:

$$\psi_\ell\left(\beta^{(i)}_\ell, u^{(i)}_\ell, \gamma_\ell, Q_\ell\right) := -\alpha_\ell \gamma_\ell (1 - q_\ell(G_\ell Q_\ell / \gamma_\ell)) + \beta^{(i)}_\ell (\kappa^{-1}_\ell Q_\ell + C_\ell), \tag{25}$$

$$\psi_{\ell+L}\left(\beta^{(i)}_\ell, u^{(i)}_\ell, \gamma_\ell, Q_\ell\right) := -1 + u^{(i)}_\ell (\kappa^{-1}_\ell Q_\ell + C_\ell). \tag{26}$$

Algorithm 1 Optimal solution of P1.

Set $\epsilon > 0$, $i = 0$, $\mathcal{C} = \epsilon + 1$.
Initialize $\boldsymbol{\beta}^{(0)}$ and $\mathbf{u}^{(0)}$.
while $\mathcal{C} > \epsilon$ **do**
Set $\lambda^* = -\min_\ell \alpha_\ell u_\ell h_\ell(x^*_{\ell,\delta_\ell=0})$, where $\forall \ell$, $x^*_{\ell,\delta_\ell=0}$ is computed as indicated in Lemma 2
If P1.1 is feasible with λ^* **then**
Find $(\mathbf{Q}^*, \boldsymbol{\gamma}^*)$ by solving P1.2
else
Find $(\mathbf{Q}^*, \boldsymbol{\gamma}^*)$ using a linesearch method (case 2 in Section IV-B)
end if
Set $\mathcal{C} = ||\boldsymbol{\psi}(\boldsymbol{\beta}^{(i)}, \mathbf{u}^{(i)}, \boldsymbol{\gamma}^*, \mathbf{Q}^*)||$.
Update $\mathbf{u}^{(i)}$ and $\boldsymbol{\beta}^{(i)}$ using the modified Newton method detailed in [30, Eqs. (33)-(34)]
$i = i + 1$.
end while

V. MGEE Problem (P2)

The MGEE problem is a fractional programming problem, which can be solved using the Dinkelbach's algorithm [47]. This iterative algorithm has been used in numerous works dealing with RA (see [14] and reference therein), and requires to solve the following problem at the ith iteration:

$$\textbf{P2.1:} \quad \max_{\mathbf{Q},\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L} \left(\alpha_\ell \gamma_\ell \left(1 - q_\ell \left(G_\ell Q_\ell / \gamma_\ell\right)\right) - \nu^{(i)} \left(\kappa_\ell^{-1} Q_\ell + C_\ell\right)\right), \quad \text{s.t.} \quad (14), (12),$$

where $\nu^{(i)} \geq 0$ depends on the optimal solution at iteration $(i-1)$. Using the same notations for the Lagrangian multipliers as for the MSEE, the KKT conditions of P2.1 write as follows

$$\alpha_\ell G_\ell q'_\ell \left(G_\ell Q_\ell / \gamma_\ell\right) (1 + \delta_\ell) + \nu^{(i)} \kappa_\ell^{-1} = 0, \quad \forall \ell, \tag{27}$$

$$\alpha_\ell h_\ell \left(G_\ell Q_\ell / \gamma_\ell\right) (1 + \delta_\ell) + \lambda = 0, \quad \forall \ell, \tag{28}$$

and the complementary slackness conditions are given by

$$\delta_\ell \left(\eta_\ell^{(0)} - \alpha_\ell \gamma_\ell \left(1 - q_\ell \left(G_\ell Q_\ell / \gamma_\ell\right)\right)\right) = 0, \quad \forall \ell, \tag{29}$$

$$\lambda \left(\sum_{\ell=1}^{L} \gamma_\ell - 1 \right) = 0. \tag{30}$$

We observe that, if $\forall \ell$ we let $u_\ell^{(i)} = 1$ and $\beta_\ell^{(i)} = \nu^{(i)}$, then the optimality conditions of the MSEE problem, i.e., (15)-(18) are the same as the ones of the MGEE problem, i.e., (27)-(30). Hence, we can apply the same procedure to solve P2.1 as the one of P1.1.

The optimal solution of P2 can be found using Algorithm 2.

Algorithm 2 Optimal solution of P2.

Set $\epsilon > 0$, $\nu^{(0)} = 0$, $i = 0$, $\mathcal{C} = \epsilon + 1$
while $|\mathcal{C}| > \epsilon_D$ **do**
Set $\lambda^* = -\min_\ell \alpha_\ell h_\ell(x^*_{\ell,\delta_\ell=0})$, where $\forall \ell$, $x^*_{\ell,\delta_\ell=0}$ is computed as indicated in Lemma 2
If P2.1 is feasible with λ^* **then**
Find $(\mathbf{Q}^*, \boldsymbol{\gamma}^*)$ by solving P1.2, where $\forall \ell$, $u_\ell^{(i)}$ is replaced by 1 and $\beta_\ell^{(i)}$ is replaced by $\nu^{(i)}$.
else
Find $(\mathbf{Q}^*, \boldsymbol{\gamma}^*)$ using a linesearch method (case 2 in Section IV-B) where $\forall \ell$, $u_\ell^{(i)}$ is replaced by 1 and $\beta_\ell^{(i)}$ is replaced by $\nu^{(i)}$.
end if
Set $\mathcal{C} = \sum_{\ell=1}^{L} \psi_\ell(\nu^{(i)}, 1, \gamma_\ell^*, Q_\ell^*)$
Update $\nu^{(i+1)} = \mathcal{G}(\mathbf{Q}^*, \boldsymbol{\gamma}^*)$
$i = i + 1$
end while

VI. MMEE Problem (P3)

The MMEE problem falls into the framework of generalized fractional programming, which can be efficiently handled using the so-called generalized Dinkelbach's algorithm [48]. This iterative algorithm has been used in the RA context for instance in [31], and requires to solve the following convex optimization problem at the ith iteration:

$$\max_{\mathbf{Q},\boldsymbol{\gamma}} \quad \min_\ell \left\{ \alpha_\ell \gamma_\ell \left(1 - q_\ell \left(G_\ell Q_\ell / \gamma_\ell\right)\right) - \psi^{(i)}(\kappa_\ell^{-1} Q_\ell + C_\ell) \right\}, \tag{31}$$

$$\text{s.t.} \quad (14), (12), \tag{32}$$

where $\psi^{(i)} \geq 0$ depends on the optimal solution at iteration $(i-1)$. We solve this problem using its epigraph formulation [49], i.e., we introduce an auxiliary optimization variable t along with the following L new constraints:

$$t \leq \alpha_\ell \gamma_\ell \left(1 - q_\ell \left(G_\ell Q_\ell / \gamma_\ell\right)\right) - \psi^{(i)}(\kappa_\ell^{-1} Q_\ell + C_\ell), \quad \forall \ell, \tag{33}$$

allowing us to rewrite it equivalently as follows:

P3.1: $\max_{\mathbf{Q}, \boldsymbol{\gamma}, t} \quad t, \quad \text{s.t.} \quad (14), (12), (33)$

This problem is the maximization of a concave function over a convex set. Defining $\boldsymbol{\omega} := [\omega_1, \ldots, \omega_L]$ as the Lagrangian multipliers associated with constraints (33) and using the same notations as in Section IV for the multipliers associated with constraints (14) and (12), the KKT conditions of P3.1 are given by

$$\sum_{\ell=1}^{L} \omega_\ell - 1 = 0, \tag{34}$$

$$\alpha_\ell G_\ell q_\ell' \left(G_\ell Q_\ell / \gamma_\ell\right) (\omega_\ell + \delta_\ell) + \omega_\ell \psi^{(i)} \kappa_\ell^{-1} = 0, \quad \forall \ell, \tag{35}$$

$$\alpha_\ell h_\ell \left(G_\ell Q_\ell / \gamma_\ell\right) (\omega_\ell + \delta_\ell) + \lambda = 0, \quad \forall \ell. \tag{36}$$

In addition, the following complementary slackness conditions hold at the optimum:

$$\omega_\ell \left(t - \alpha_\ell \gamma_\ell \left(1 - q_\ell \left(G_\ell Q_\ell / \gamma_\ell\right)\right) + \psi^{(i)} \left(\kappa_\ell^{-1} Q_\ell + C_\ell\right)\right) = 0, \quad \forall \ell, \tag{37}$$

$$\delta_\ell \left(\eta_\ell^{(0)} - \alpha_\ell \gamma_\ell \left(1 - q_\ell \left(G_\ell Q_\ell / \gamma_\ell\right)\right)\right) = 0, \quad \forall \ell, \tag{38}$$

$$\lambda \left(\sum_{\ell=1}^{L} \gamma_\ell - 1\right) = 0. \tag{39}$$

We observe an important difference between the KKT conditions related to P3.1 as compared with the ones of P1.1 and P2.1: $\forall \ell$, the optimality condition (36) involves three distinct Lagrangian multipliers, λ, ω_ℓ and δ_ℓ, preventing us from expressing the optimal solution of P3.1 as a function of a single multiplier. Fortunately, in the following lemma whose proof is provided in Appendix C, we are able to prove that constraints (12) and (33) hold with equality.

Lemma 3. *At the optimum of P3.1,* $\lambda > 0$ *and,* $\forall \ell$*,* $\omega_\ell > 0$*.*

Since $\lambda > 0$, the conditions (35) and (36) can be rewritten as follows:

$$\alpha_\ell G_\ell q_\ell' \left(G_\ell Q_\ell / \gamma_\ell\right) (\tilde{\omega}_\ell + \tilde{\delta}_\ell) + \tilde{\omega}_\ell \psi^{(i)} \kappa_\ell^{-1} = 0, \quad \forall \ell, \tag{40}$$

$$\alpha_\ell h_\ell \left(G_\ell Q_\ell/\gamma_\ell\right)(\tilde{\omega}_\ell + \tilde{\delta}_\ell) + 1 = 0, \quad \forall \ell, \tag{41}$$

with, $\forall \ell$, $\tilde{\omega}_\ell := \omega_\ell/\lambda$ and $\tilde{\delta}_\ell := \delta_\ell/\lambda$.

Thanks to Lemma 3, we can use tools from the multilevel waterfilling theory [52] to find the optimal solution of P3.1. The idea is to express parameters $x_\ell := G_\ell Q_\ell/\gamma_\ell$ (which is equal to $G_\ell E_\ell$) and γ_ℓ as a function of the single parameter t using (37). The condition (39) is then used to obtain the optimal value of t, enabling us to find the optimal values of γ_ℓ and x_ℓ, and as a consequence the optimal Q_ℓ and then E_ℓ.

Let us define $\tilde{\boldsymbol{\omega}} := [\tilde{\omega}_1, \ldots, \tilde{\omega}_L]$. We also define I_t (resp. $\bar{I}_t$) as the set of links with $\tilde{\delta}_\ell = 0$ (resp. $\tilde{\delta}_\ell > 0$). In the following, we first consider $\tilde{\boldsymbol{\omega}}$ as fixed, and we find the optimal values of x_ℓ and γ_ℓ for the links in I_t and $\bar{I}_t$ as a function of t, as well as a characterization of these two sets.

A. *Solution for Fixed $\tilde{\boldsymbol{\omega}}$ and t*

Case 1: $\ell \in I_t$. From (40), we obtain $x^*_{\ell,1}$, the optimal value of x_ℓ, as follows:

$$x^*_{\ell,1} = q'^{-1}_\ell \left(\frac{-\psi^{(i)}\kappa_\ell^{-1}}{\alpha_\ell G_\ell}\right). \tag{42}$$

Using Lemma 3 and (37), we obtain $\gamma^*_{\ell,1}(t)$, the optimal value of γ_ℓ, depending only on t as:

$$\gamma^*_{\ell,1}(t) = \frac{t + \psi^{(i)}C_\ell}{\alpha_\ell(1 - q_\ell(x^*_{\ell,1})) - \psi^{(i)}\kappa_\ell^{-1}G_\ell^{-1}x^*_{\ell,1}}. \tag{43}$$

The following lemma, whose proof is provided in Appendix D, enables us to check whether ℓ belongs to I_t or not.

Lemma 4. *A link ℓ is in I_t iff the following inequality holds:*

$$t \geq t_\ell^T, \tag{44}$$

with $t_\ell^T := -\psi^{(i)}C_\ell + \eta_\ell^{(0)}(1 - (\psi^{(i)}\kappa_\ell^{-1}G_\ell^{-1}x^*_{\ell,1})/(\alpha_\ell(1 - q_\ell(x^*_{\ell,1}))))$.

Case 2: $\ell \in \bar{I}_t$.

1) Optimal solution as a function of $\tilde{\omega}_\ell$: Similarly to the derivations related to P1.1, using (40) and (41) we obtain $x^*_{\ell,2}(\tilde{\omega}_\ell)$, the optimal x_ℓ, as follows:

$$x^*_{\ell,2}(\tilde{\omega}_\ell) := g_\ell^{-1}\left(\frac{1}{\tilde{\omega}_\ell \psi^{(i)}\kappa_\ell^{-1}}\right). \tag{45}$$

Since $\tilde{\delta}_\ell > 0$, we obtain from (38) $\gamma^*_{\ell,2}(\tilde{\omega}_\ell)$, the optimal γ_ℓ, depending only on $\tilde{\omega}_\ell$ as:

$$\gamma^*_{\ell,2}(\tilde{\omega}_\ell) = \frac{\eta_\ell^{(0)}}{\alpha_\ell(1 - q_\ell(x^*_{\ell,2}(\tilde{\omega}_\ell)))}. \tag{46}$$

We have managed to obtain the optimal values of x_ℓ and γ_ℓ for fixed $\tilde{\boldsymbol{\omega}}$ and t. Now, we turn our attention to exhibit a relation between $\tilde{\omega}_\ell$ and t in order to express $x^*_{\ell,2}(\tilde{\omega}_\ell)$ and $\gamma^*_{\ell,2}(\tilde{\omega}_\ell)$ as function of t.

2) Relation between $\tilde{\omega}_\ell$ and t: Using Lemma 3, we obtain the following L relations by plugging (45) and (46) into (37):

$$t = \mathcal{M}_\ell(\tilde{\omega}_\ell), \quad \forall \ell, \tag{47}$$

with $\omega \mapsto \mathcal{M}_\ell(\omega) := \eta_\ell^{(0)} - \psi^{(i)}(\kappa_\ell^{-1}\alpha_\ell^{-1}x^*_{\ell,2}(\omega)/(1 - q_\ell(x^*_{\ell,2}(\omega))) + C_\ell)$. To express $\tilde{\omega}_\ell$ as a function of t, we use the following lemma, whose proof is provided in Appendix E.

Lemma 5. *$\forall \ell$, the function $\mathcal{M}_\ell$ is continuous and strictly increasing, and thus $\mathcal{M}_\ell^{-1}$ exists and is strictly increasing.*

Using Lemma 5 in conjunction with (47) yields

$$\tilde{\omega}_\ell = \mathcal{M}_\ell^{-1}(t), \quad \forall \ell, \tag{48}$$

and then we can obtain $\gamma^*_{\ell,2}$ as a function of t by plugging (48) into (46). As a consequence, $\gamma^*_{\ell,2}(\mathcal{M}_\ell^{-1}(t))$, shortened to $\gamma^*_{\ell,2}(t)$ by abuse of notation, is given by:

$$\gamma^*_{\ell,2}(t) = \frac{\eta_\ell^{(0)}}{\alpha_\ell(1 - q_\ell(x^*_{\ell,2}(\mathcal{M}_\ell^{-1}(t))))}. \tag{49}$$

For a given t, we have succeeded to find a necessary and sufficient condition given in Lemma 4 to check whether a node belongs to I_t or $\bar{I}_t$, and we have found the optimal parameters in both cases. Now we search for the optimal value of t.

B. Search for the Optimal t

To find t^*, the optimal value of t, we use the complementary slackness condition (39). Let us define the following function representing the sum of the bandwidth parameters for given value of t

$$\tilde{\Gamma}(t) := \sum_{\ell \in I_t} \gamma^*_{\ell,1}(t) + \sum_{\ell \in \bar{I}_t} \gamma^*_{\ell,2}(t). \tag{50}$$

Due to (39), t^* is such that $\tilde{\Gamma}(t^*) = 1$. In the following lemma whose proof is provided in Appendix F, we prove that such a t^* always exists, and can be found through a linesearch.

Lemma 6. *The function $\tilde{\Gamma}(t)$ is continuous, strictly decreasing, and there exists t^* such that $\tilde{\Gamma}(t^*) = 1$.*

The optimal solution of P3.1 can be found by solving $\tilde{\Gamma}(t^*) = 1$, which always has a solution. Then, the optimal values $x^*_{\ell,i}(t^*)$ and $\gamma^*_{\ell,i}(t^*)$, $i \in \{1, 2\}$, are computed, and we deduce the optimal $Q^*_\ell(t^*)$.

C. Optimal Solution of P3

The optimal solution of P3 can be found using Algorithm 3.

Algorithm 3 Optimal solution of P3.

Set $\epsilon > 0$, $\psi^{(0)} = 0$, $i = 0$, $t^* = \epsilon + 1$
while $t^* > \epsilon$ **do**
Compute t^*, $\mathbf{Q}^*$ and $\boldsymbol{\gamma}^*$ by solving P3.1 with $\psi^{(i)}$.
Update $\psi^{(i+1)} = \min_{\ell \in \{1,\dots,L\}} \mathcal{E}_\ell(Q^*_\ell/\gamma^*_\ell, \gamma^*_\ell)$.
$i = i + 1$.
end while

VII. Numerical Examples

In this section, the results of the proposed algorithms are numerically studied. First, we compare the performance of the proposed criteria under both the Rayleigh and the Rician channel. Second, we investigate the benefits of considering the Rician channel instead of the conventional Rayleigh channel. Third, we study the impact of the circuitry power consumption on the MSEE performance. Finally, we study possible extension of the proposed criterion to Type-II HARQ. We also compare the proposed criteria with a conventional one from the literature: the minimum power (MPO) from [44] minimizing the total transmit power.

We use a convolutional code with rate $1/2$ with generator polynomial $[171, 133]_8$, and we use the quadrature phase shift keying (QPSK) modulation, i.e., $m_\ell = 2$. The number of links is $L = 5$, and the link distances D_ℓ are uniformly drawn in $[50 \text{ m}, 1 \text{ km}]$. We consider that all the links have an identical K factor value. Unless otherwise stated, we simulate both Rician channel with $K_\ell = 10$ and Rayleigh channel ($K_\ell = 0$). We set $B = 5$ MHz, $N_0 = -170$ dBm/Hz and $\mathcal{L}_\ell = 128$. The carrier frequency is $f_c = 2400$ MHz and we put $\zeta_\ell^2 = (4\pi f_c/c)^{-2} D_\ell^{-3}$ where c is the speed of light in vacuum. We assume that the per-link minimum goodput $\eta_\ell^{(t)}$ is equal for all links. Except otherwise stated, we put $\forall \ell$, $P_{ctx,\ell} = P_{crx,\ell} = 0.05$ W and $\kappa_\ell = 0.5$. We perform RA using the PER approximation (10), whose accuracy is validated through simulations in [44, Figs. 1 and 2].

A. *Performance of the Proposed Algorithms*

In Figs. 1-3, we plot the SEE, MEE and GEE obtained with the proposed criteria and with the MPO from [44] versus the per-link minimum goodput constraint $\eta_\ell^{(t)}$. We perform optimal RA according to the links channel distribution: Rayleigh RA under Rayleigh channel and Rician RA under Rician channel. As expected, the maximization of a given criterion yields the highest value for this criterion. The proposed criteria yield higher EE than the conventional MPO, especially for low goodput constraint. Due to Type-I HARQ EE expression, one can check that EE under the Rician channel is higher than those under the Rayleigh channel because of better PER for the Rician channel. We actually observe that the performance under the Rician channel is *much* higher than those under the Rayleigh channel.

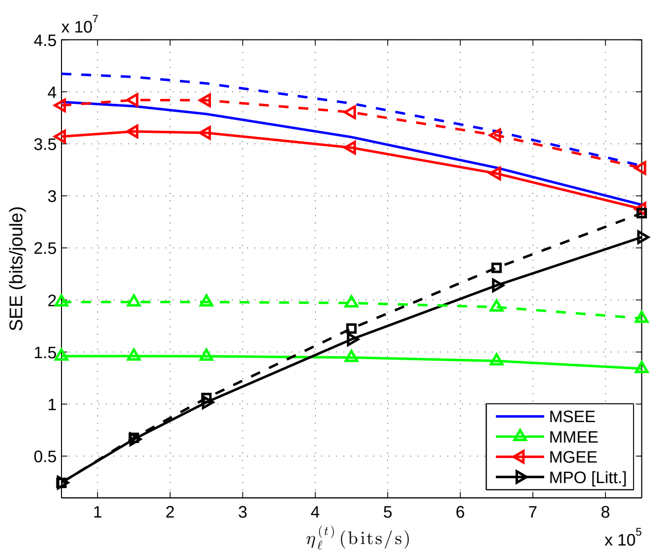

Fig. 1. SEE obtained for the considered criteria versus $\eta_\ell^{(t)}$, solid lines: Rayleigh channel ($\forall \ell$, $K_\ell = 0$), dashed lines: Rician channel ($\forall \ell$, $K_\ell = 10$).

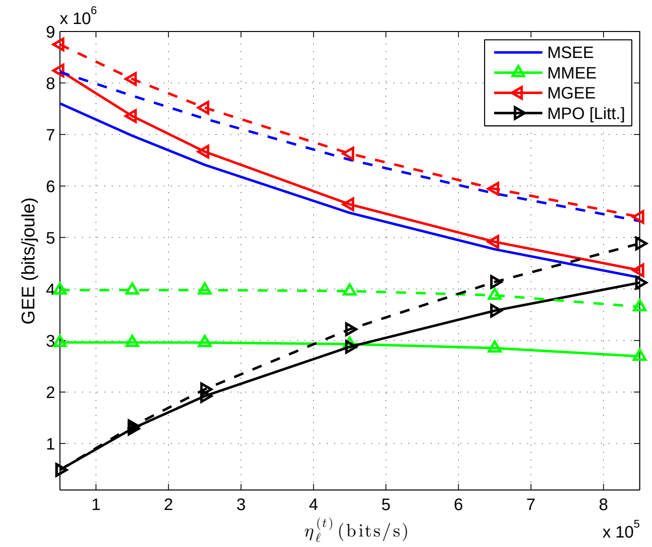

Fig. 2. GEE obtained for the considered criteria versus $\eta_\ell^{(t)}$, solid lines: Rayleigh channel ($\forall \ell$, $K_\ell = 0$), dashed lines: Rician channel ($\forall \ell$, $K_\ell = 10$).

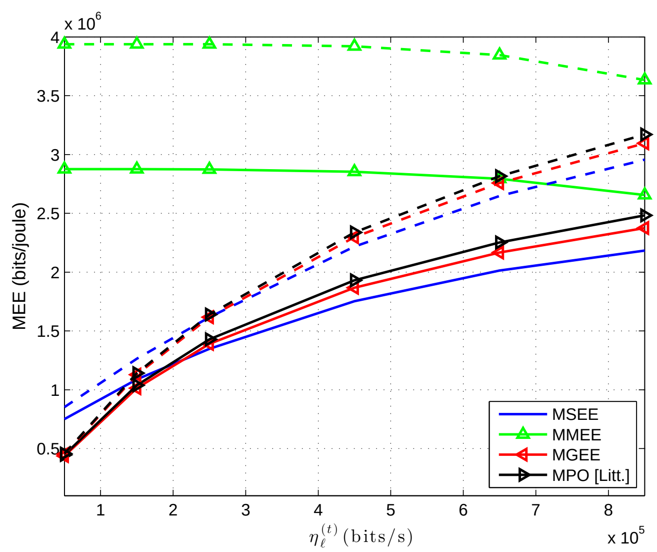

Fig. 3. MEE obtained for the considered criteria versus $\eta_\ell^{(t)}$, solid lines: Rayleigh channel ($\forall \ell$, $K_\ell = 0$), dashed lines: Rician channel ($\forall \ell$, $K_\ell = 10$).

We would like now to analyze a mismatch between the value of K_ℓ used for calculating RA and its actual value for the same GNR. More precisely, we consider the Rician channel with various values of K_ℓ as the actual channel statistics. We calculate RA either *i*) by assuming a Rayleigh channel, i.e., $K_\ell = 0$ (channel model mismatch case) or *ii*) by assuming a Rician channel with the true value of K_ℓ (no channel model mismatch case). We thus define $\mathbf{E}_R^*$ and γ_R^* as the optimal values of $\mathbf{E}$ and γ when the Rician channel model is considered. We also define $\mathbf{E}_C^*$ and γ_C^* as the optimal values of $\mathbf{E}$ and γ when the Rayleigh channel is considered. We can thus define the EE gains between the Rician and the Rayleigh allocations under the Rician channel as $100 \times (\mathcal{Z}_R(\mathbf{E}_R^*, \gamma_R^*) - \mathcal{Z}_R(\mathbf{E}_C^*, \gamma_C^*))/\mathcal{Z}_R(\mathbf{E}_C^*, \gamma_C^*)$, where $\mathcal{Z}_R$ stands for either the SEE, the MEE or the GEE calculated under the Rician channel with the actual value of K_ℓ.

In Fig. 4, we assume that the actual channel statistic is $K_\ell = 10$, and we plot the EE gains versus the minimum goodput constraint. We observe that substantial gains can be achieved for all the criteria, especially for the MMEE.

In Fig. 5, we set the minimum goodput constraint $\eta_\ell^{(t)} = 4.5 \times 10^5$ bits/s, and we plot the EE gains versus the actual value of K_ℓ. Once again, we observe that substantial gains can be achieved for all the criteria. We also observe that this gain is strictly increasing with the value of K_ℓ.

The achieved gains are due to the following key point which have been checked through simulation but is not included due to space limitation: the EE maximizer is different under the Rayleigh and the Rician channel. It is thus highly beneficial to incorporate the knowledge of the Rician K-factor in RA.

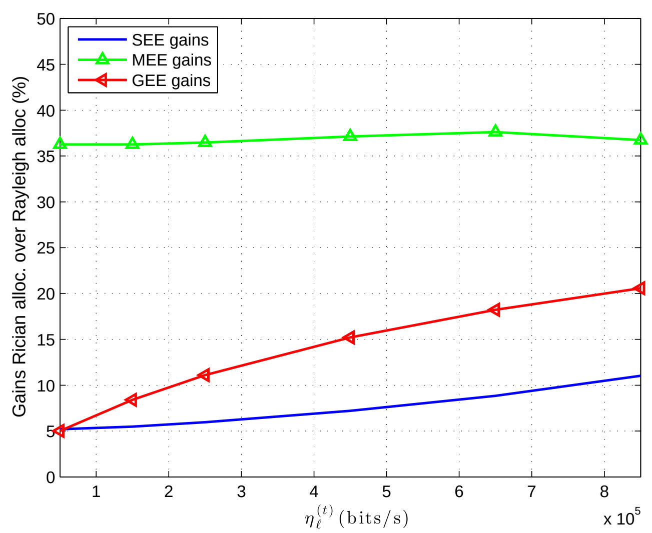

Fig. 4. Gains between the Rician ($\forall \ell, K_\ell = 10$) and the Rayleigh ($\forall \ell, K_\ell = 0$) allocations under the Rician channel, versus $\eta_\ell^{(t)}$.

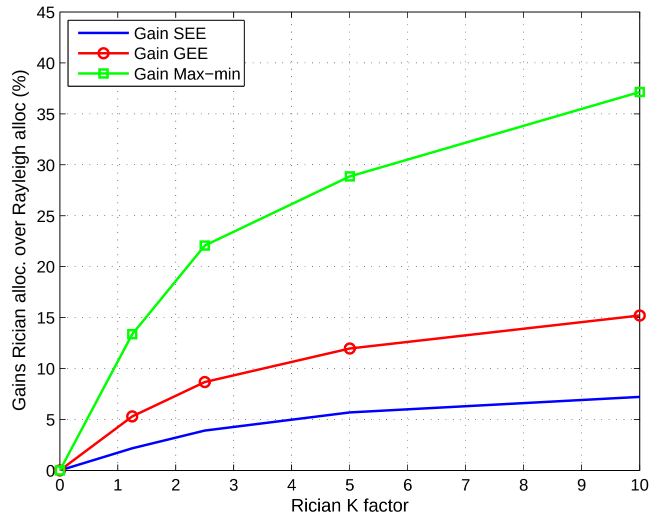

Fig. 5. Gains between the Rician ($\forall \ell, K_\ell = 10$) and the Rayleigh ($\forall \ell, K_\ell = 0$) allocations under the Rician channel, versus the Rician K factor, with $\eta_\ell^{(t)} = 4.5 \times 10^5$ bits/s.

B. Influence of P_{ctx} and P_{crx}

In this section, we investigate the performance of the circuitry power consumptions $P_{ctx,\ell}$ and $P_{crx,\ell}$. For sake of simplicity, we assume $P_{ctx,\ell} = P_{crx,\ell}, \forall \ell$ and we denote $P_c := P_{ctx,\ell}$. We only study the MSEE criterion since the trends are the same for all the criteria, and thus they do not provide further insights. In Fig. 6, we set the per-link minimum goodput constraint $\eta_\ell^{(t)} = 1.25 \times 10^5$ bits/s, and we plot the SEE versus the circuitry power consumption for Rician channel (Algorithm 1) and for Rayleigh channel [43]. We see that the SEE performance decreases as P_c increases, which is in agreement with the intuition. We also observe that the gap between Rician channel and Rayleigh channel vanishes when the circuitry consumption goes to infinity since the impact of the channel type decreases when the circuitry consumption is dominant.

C. Application to Type-II HARQ

From Table I, we see that there is no work addressing EE-based RA for Type-II HARQ under the Rician channel and thus we look for solutions to these problems. We did not succeed to optimally solve these problems for the following technical reasons. Let us assume a Type-II HARQ mechanism for link ℓ using the same packet length. According to [43], the EE for link

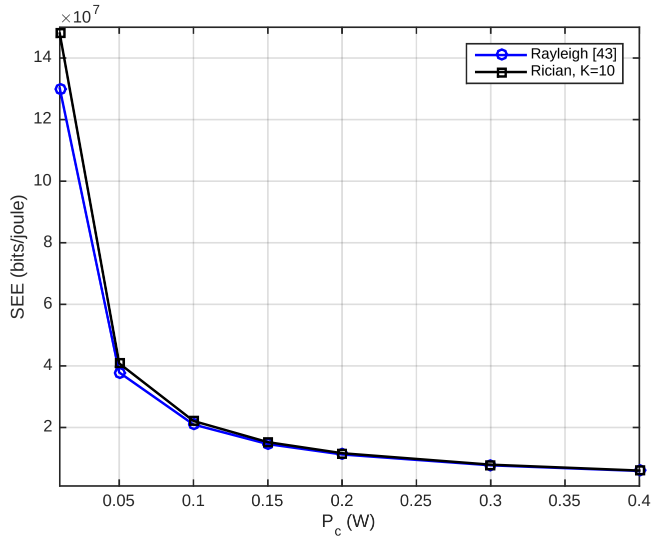

Fig. 6. Optimal SEE under both the Rayleigh ($\forall \ell, K_\ell = 0$) and the Rician channel ($\forall \ell, K_\ell = 10$) versus P_c, with $\eta_\ell^{(t)} = 1.25 \times 10^5$ bits/s.

ℓ can be written as

$$\tilde{\mathcal{E}}_\ell(E_\ell, \gamma_\ell) = \frac{\alpha_\ell \gamma_\ell (1 - q_{\ell,\mathcal{T}}(E_\ell G_\ell))}{(1 + \sum_{k=1}^{\mathcal{T}-1} q_{\ell,k}(E_\ell G_\ell))(\kappa_\ell^{-1} \gamma_\ell E_\ell + E_{c,\ell})}, \tag{51}$$

where $q_{\ell,k}$ is the probability that the first k transmissions are all received in error which occurs in Type-II HARQ since we keep the packets received in error to help the decoding. By comparing (8) and (51), we see that the difference between Type-I and Type-II HARQ EE lies in the presence of $q_{\ell,k}$, $k > 1$ at the denominator of (51). By applying the change of variables given by (13) into (51), we obtain

$$\tilde{\mathcal{E}}_\ell(Q_\ell, \gamma_\ell) = \frac{\alpha_\ell \gamma_\ell (1 - q_{\ell,\mathcal{T}}(G_\ell Q_\ell / \gamma_\ell))}{(1 + \sum_{k=1}^{\mathcal{T}-1} q_{\ell,k}(G_\ell Q_\ell / \gamma_\ell))(\kappa_\ell^{-1} Q_\ell + E_{c,\ell})}. \tag{52}$$

Unless $\mathcal{T} = 1$, we cannot identify a property ensuring that the denominator of (52) is a concave function and thus the tools from fractional programming theory cannot be applied with affordable complexity to solve EE based RA problems. In [43] (where only Rayleigh channel is considered with an approximation different to (10) but not suitable for Rician channel), the authors overcome this issue as the denominator was posynomial and so geometric programming tools have been employed to optimally solve the MSEE problem. In (52), the denominator is not posynomial preventing us from applying the approach proposed in [43].

Therefore, we look for suboptimal solutions by extending the results from [43] and from the results related to Type-I HARQ of this paper. As [43] provides the MSEE optimal solution for Type-II HARQ under the Rayleigh channel, we only focus on this criterion. When Type-II HARQ is simulated, the SEE is given by

$$S(\mathbf{E}, \boldsymbol{\gamma}) := \sum_{\ell=1}^{L} \tilde{\mathcal{E}}_\ell(E_\ell, \gamma_\ell).$$

To that end, we consider the following two manners as illustrated in Fig. 7.

- Channel model mismatch: using the resources from Type-II HARQ Rayleigh RA (i.e., the solution from [43]) on Type-II HARQ system under the Rician channel. These resources are denoted by $\mathbf{E}^*_{\mathrm{II}}$ and $\boldsymbol{\gamma}^*_{\mathrm{II}}$.
- HARQ type mismatch: using the resources from Type-I HARQ Rician RA (i.e., the solution from Algorithm 1) on Type-II HARQ system under the Rician channel. These resources are denoted by $\mathbf{E}^*_{\mathrm{I}}$ and $\boldsymbol{\gamma}^*_{\mathrm{I}}$.

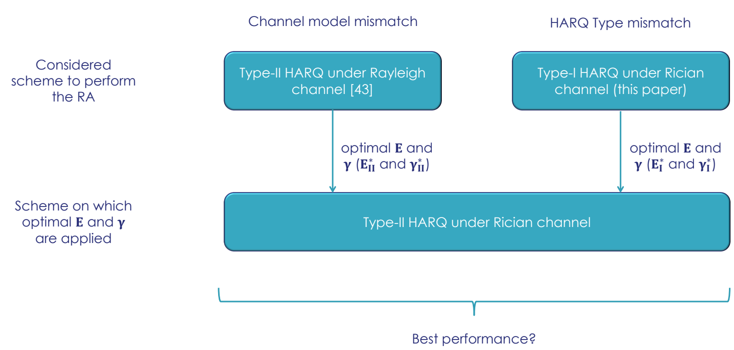

Fig. 7. The two considered solutions to extend either the work from [43] or from this paper to Type-II HARQ under the Rician channel.

In Fig. 8, we simulate a chase combining (CC) HARQ mechanism with maximum number of transmissions $\mathcal{T} = 3$ under a Rician channel with $\forall \ell$, $K_\ell = 10$ and we plot $S(\mathbf{E}^*_{\mathrm{I}}, \gamma^*_{\mathrm{I}})$ and $S(\mathbf{E}^*_{\mathrm{II}}, \gamma^*_{\mathrm{II}})$ versus the per-link minimum goodput constraint $\eta_\ell^{(t)}$. We observe that $S(\mathbf{E}^*_{\mathrm{I}}, \gamma^*_{\mathrm{I}})$ is much higher than $S(\mathbf{E}^*_{\mathrm{II}}, \gamma^*_{\mathrm{II}})$ whatever the goodput constraint, yielding us to advocate that the algorithm is less sensitive to HARQ type mismatch rather than to channel model mismatch.

VIII. Conclusion

In this paper, we addressed the problem of joint bandwidth and power allocation for Type-I HARQ with practical MCS and statistical CSI under the Rician channel by providing the analytical optimal solutions of three RA problems: the MSEE, the MMEE and the GEE. Through

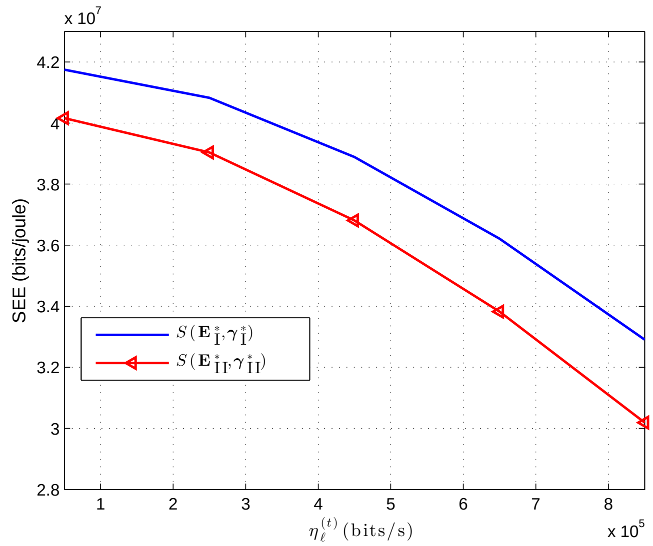

Fig. 8. SEE obtained using MSEE algorithm for Type-I and Type-II HARQ under the Rician channel ($\forall \ell, K_\ell = 10$) versus $\eta_\ell^{(t)}$.

simulations, we exhibited that substantial gains can be achieved by taking into account the existence of a LoS between the transmitter and the receiver instead of only considering the channel average power. We also studied the behavior of our proposed algorithms when applied on Type-II HARQ. Numerical results indicated that, when the channel is Rician, it is more beneficial to apply RA corresponding to Type-I HARQ for the Rician channel instead of the Type-II RA for the Rayleigh channel.

Appendix A
Proof of Lemma 1

First, let us prove that the feasible set defined by constraints (12) and (14) is convex. Constraint (12) is linear and as a consequence it is convex. Moreover, $\gamma_\ell(1 - q_\ell(G_\ell Q_\ell/\gamma_\ell))$ is the so called perspective [49] of the concave function $1 - q_\ell(G_\ell E_\ell)$ and thus it is concave, meaning that constraint (14) is convex.

Second, let us focus on the objective functions of P1'-P3'. We remark that there denominators are linear and thus they are convex. The numerators of the objective functions of P1' and P2' are given by $\alpha_\ell\gamma_\ell(1 - q_\ell(G_\ell Q_\ell/\gamma_\ell))$ and hence they are concave as the perspective of concave functions. For P3', the numerator is given by $\sum_{\ell=1}^{L} \alpha_\ell(\gamma_\ell(1-q_\ell(G_\ell Q_\ell/\gamma_\ell)))$ and then it is concave as a sum of concave functions.

Appendix B
Proof of Lemma 2

First due to (16), we are only interested in solutions yielding non positive values for h_ℓ. If there exists at least one link ℓ_1 with $\delta_{\ell_1} = 0$, we obtain the optimal value of x_{ℓ_1} using (19) as:

$$x^*_{\ell_1} = x^*_{\ell_1,\delta_{\ell_1}=0} = q'^{-1}_{\ell_1}\left(\frac{-\beta^{(i)}_{\ell_1}\kappa^{-1}_{\ell_1}}{\alpha_{\ell_1}G_{\ell_1}}\right). \tag{53}$$

By plugging (53) into (16), we obtain the optimal value of λ as:

$$\lambda^* = -\alpha_{\ell_1}u^{(i)}_{\ell_1}h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0}) \geq 0. \tag{54}$$

Hence, we prove that $\ell_1 \in \arg\min_\ell\{\alpha_\ell u^{(i)}_\ell h_\ell(x_{\ell,\delta_\ell=0})\}$. To do so, we proceed by contradiction: we assume that $\exists \ell_2$ such that $\alpha_{\ell_2}u^{(i)}_{\ell_2}h_{\ell_2}(x^*_{\ell_2,\delta_{\ell_2}=0}) < \alpha_{\ell_1}u^{(i)}_{\ell_1}h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0})$, and we prove that the KKT condition (16) cannot hold for ℓ_2. This condition writes as follows:

$$\alpha_{\ell_2}u^{(i)}_{\ell_2}h_{\ell_2}(x^*_{\ell_2})(u^{(i)}_{\ell_2}+\delta_{\ell_2}) - \alpha_{\ell_1}u^{(i)}_{\ell_1}h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0}) = 0. \tag{55}$$

To prove that (55) cannot hold, we upper bound it by a term strictly lower than 0. To this end, we use the following proposition.

Proposition 1. *$\forall \ell$, the following inequality holds:*

$$h_\ell(x^*_\ell) \leq h_\ell(x^*_{\ell,\delta_\ell=0}). \tag{56}$$

Proof. First, let us study the monotonicity of $h_\ell(x)$ by computing its first order derivative:

$$h'_\ell(x) = -x q''_\ell(x). \tag{57}$$

Due to the strict convexity of q_ℓ, it results from (57) that h_ℓ is strictly decreasing.

Second, let us compare x^*_ℓ with $x^*_{\ell,\delta_\ell=0}$. From (19), we have

$$x^*_\ell = q'^{-1}_\ell\left(\frac{-u^{(i)}_\ell\beta^{(i)}_\ell\kappa^{-1}_\ell}{\alpha_\ell G_\ell(u^{(i)}_\ell+\delta_\ell)}\right). \tag{58}$$

Since q'^{-1}_ℓ is increasing, the following inequality holds: $x^*_\ell \geq x^*_{\ell,\delta_\ell=0}$. Finally, the proof is completed using (57). □

Using Proposition 1 we can upper bound (55) as follows

$$\begin{aligned}&\alpha_{\ell_2}u^{(i)}_{\ell_2}h_{\ell_2}(x^*_{\ell_2})(u^{(i)}_{\ell_2}+\delta_{\ell_2}) - \alpha_{\ell_1}u^{(i)}_{\ell_1}h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0}) \leq \\ &\alpha_{\ell_2}u^{(i)}_{\ell_2}h_{\ell_2}(x^*_{\ell_2,\delta_{\ell_2}=0})(u^{(i)}_{\ell_2}+\delta_{\ell_2}) - \alpha_{\ell_1}u^{(i)}_{\ell_1}h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0}).\end{aligned} \tag{59}$$

Since by hypothesis, $\alpha_{\ell_2} u_{\ell_2} h_{\ell_2}(x^*_{\ell_2,\delta_{\ell_2}=0}) < \alpha_{\ell_1} u_{\ell_1} h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0})$, $\alpha_{\ell_1} u_{\ell_1} h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0}) = -\lambda^* \leq 0$ and $\delta_{\ell_2} \geq 0$, we obtain from (59)

$$\alpha_{\ell_2} u^{(i)}_{\ell_2} h_{\ell_2}(x^*_{\ell_2})(u^{(i)}_{\ell_2} + \delta_{\ell_2}) - \alpha_{\ell_1} u^{(i)}_{\ell_1} h_{\ell_1}(x^*_{\ell_1,\delta_{\ell_1}=0}) < 0. \tag{60}$$

Due to (60), the KKT condition (16) cannot hold for link ℓ_2 yielding a contradiction.

APPENDIX C
PROOF OF LEMMA 3

First, let us prove that $\lambda > 0$. We need the following intermediate result.

Proposition 2. *At any iteration i, the optimal t is such that $t \geq 0$.*

Proof. The proof is similar to the one of [48, Proposition 2] and is thus omitted. □

The rest of the proof is by contradiction: we assume that $\lambda = 0$, and we prove that it yields a strictly negative value for t, which contradicts Proposition 2. To do so, we remark from (34) that $\sum_{\ell=1}^{L} \omega_\ell = 1$, meaning that $\exists \ell$ such that $\omega_\ell > 0$. Let us focus on this link. We consider the following two possible cases: either $\delta_\ell = 0$ or $\delta_\ell > 0$.

Case 1: $\delta_\ell = 0$. Using (35) and (36), we obtain:

$$\alpha_\ell(q_\ell(x_\ell) - 1) + \frac{x_\ell \kappa_\ell^{-1} \psi^{(i)}}{G_\ell} = 0, \tag{61}$$

with $x_\ell := G_\ell Q_\ell / \gamma_\ell$. In addition, since $\omega_\ell > 0$, plugging (61) into (37) yields

$$t = \gamma_\ell \left(\alpha_\ell \left(1 - q_\ell(x_\ell)\right) - \frac{x_\ell \psi^{(i)} \kappa_\ell^{-1}}{G_\ell} \right) - \psi^{(i)} C_\ell = -\psi^{(i)} C_\ell < 0. \tag{62}$$

Case 2: $\delta_\ell > 0$. The condition (35) gives us:

$$\alpha_\ell(-1 + q_\ell(x_\ell) - x_\ell q'_\ell(x_\ell)) = 0. \tag{63}$$

Since $\delta_\ell > 0$, we obtain:

$$\gamma_\ell = \frac{\eta_\ell^{(0)}}{\alpha_\ell(1 - q_\ell(x_\ell))}. \tag{64}$$

By plugging (63) and (64) into (37), we obtain

$$t = \eta_\ell^{(0)} \left(1 + \frac{\psi^{(i)} \kappa_\ell^{-1}}{\alpha_\ell G_\ell q'_\ell(x_\ell)} \right) - \psi^{(i)} C_\ell. \tag{65}$$

To upper bound (65), we use (36) which gives us

$$\frac{\psi^{(i)} \kappa_\ell^{-1}}{\alpha_\ell G_\ell q'_\ell(x_\ell)} < -1. \tag{66}$$

Using (66) into (65) yields $t < 0$.

Gathering case 1 and case 2 together, we obtain that $\lambda = 0$ yields $t < 0$, contradicting Proposition 2. Hence, we deduce that $\lambda > 0$.

Now, let us prove that, $\forall \ell$, $\omega_\ell > 0$. Assume that there exists ℓ such that $\omega_\ell = 0$. We can see from (35) that $\delta_\ell = 0$. However, plugging $\omega_\ell = 0$ and $\delta_\ell = 0$ into (36) implies $\lambda = 0$, which contradicts $\lambda > 0$. Hence, $\forall \ell$, $\omega_\ell > 0$ which concludes the proof.

Appendix D
Proof of Lemma 4

First, assume that a link ℓ belongs to I_t. Its optimal values for x_ℓ and γ_ℓ are given by (42) and (43), respectively. Moreover, link ℓ has to satisfy its goodput constraint (14). By plugging (42) and (43) into (14), the direct part of Lemma 4 is proved.

Second, we prove the converse part of Lemma 4 by contradiction: assuming that there exists a link ℓ such that inequality (44) holds and which is not in I_t, we prove that the optimality condition (38) cannot hold. Let us define $x^*_{\ell,2}(\delta_\ell)$ (resp. $\gamma^*_{\ell,2}(\delta_\ell, t)$) the optimal value of x_ℓ (resp. γ_ℓ) for fixed ω_ℓ and t. Notice that $x^*_{\ell,2}(0)$ (resp. $\gamma_{\ell,2}(0,t)$) coincides with $x^*_{\ell,1}$ (resp. $\gamma^*_{\ell,1}(t)$). With these notations, (44) can be rewritten as follows:

$$\eta_\ell^{(0)} \leq \alpha_\ell \gamma^*_{\ell,2}(0,t)(1 - q_\ell(x^*_{\ell,2}(0))). \tag{67}$$

Since $\delta_\ell > 0$, (38) yields:

$$\eta_\ell^{(0)} = \alpha_\ell \gamma^*_{\ell,2}(\delta_\ell,t)(1 - q_\ell(x^*_{\ell,2}(\delta_\ell))). \tag{68}$$

To show the contradiction, we prove that (68) cannot hold using the following proposition.

Proposition 3. *For all $\delta_\ell > 0$ and $\forall \ell$, the following inequalities hold:*

$$x^*_{\ell,2}(\delta_\ell) > x^*_{\ell,2}(0) \tag{69}$$

$$\gamma^*_{\ell,2}(\delta_\ell, t) > \gamma^*_{\ell,2}(0, t) \tag{70}$$

Proof. We start by proving (69). Using (41), we obtain

$$x^*_{\ell,2}(\delta_\ell) = q_\ell'^{-1}\left(\frac{-\omega_\ell \psi^{(i)} \kappa_\ell^{-1}}{\alpha_\ell G_\ell(\omega_\ell + \delta_\ell)}\right). \tag{71}$$

Since $q_\ell'^{-1}$ is continuous, differentiable with non zero derivative and strictly increasing, $x^*_{\ell,2}(\delta_\ell)$ is a continuous, differentiable and strictly increasing function of δ_ℓ, which proves (69).

Now, let us focus on $\gamma^*_{\ell,2}(\delta_\ell, t)$. Using Lemma 3, we can obtain:

$$\gamma^*_{\ell,2}(\delta_\ell, t) = \frac{t + \psi^{(i)} C_\ell}{f_\ell(\delta_\ell)}, \tag{72}$$

with $f_\ell(\delta_\ell) := \alpha_\ell(1 - q_\ell(x^*_{\ell,2}(\delta_\ell)) - \psi^{(i)}\kappa_\ell^{-1} G_\ell^{-1} x^*_{\ell,2}(\delta_\ell)$. To prove (70), let us prove that $f_\ell(\delta_\ell)$ is strictly decreasing by computing its derivative:

$$f'_\ell(\delta_\ell) = -x^{*'}_{\ell,2}(\delta_\ell) g_\ell(\delta_\ell), \tag{73}$$

with $x^{*'}_{\ell,2}(\delta_\ell) > 0$ the derivative of $x^*_{\ell,2}(\delta_\ell)$ with respect to δ_ℓ, and $g_\ell(\delta_\ell) := (\alpha_\ell q'_\ell(x^*_{\ell,2}(\delta_\ell)) + \psi^{(i)}\kappa_\ell^{-1} G_\ell^{-1})$. Using (71), we can see that $g_\ell(0) = 0$, meaning that $f'_\ell(0) = 0$. In addition, we can prove that $g_\ell(\delta_\ell)$ is strictly increasing by computing its derivative, meaning that, for all $\delta_\ell > 0$, $g_\ell(\delta_\ell) > 0$ which, together with (73) concludes the proof. □

Using Proposition 3, we hence have, for all $\delta_\ell > 0$:

$$\alpha_\ell \gamma^*_{\ell,2}(\delta_\ell, t)(1 - q_\ell(x^*_{\ell,2}(\delta_\ell))) > \alpha_\ell \gamma^*_{\ell,2}(0, t)(1 - q_\ell(x^*_{\ell,2}(0))) \geq \eta_\ell^{(0)}, \tag{74}$$

which contradicts (68) and concludes the proof.

APPENDIX E
PROOF OF LEMMA 5

It is sufficient to prove that $\mathcal{F}_\ell(\omega_\ell) := x^*_{\ell,2}(\omega_\ell)/(1 - q_\ell(x^*_{\ell,2}(\omega_\ell)))$ is strictly decreasing. Let us compute the derivative of $\mathcal{F}_\ell(\omega_\ell)$:

$$\mathcal{F}'_\ell(\omega_\ell) = -\frac{x^{*'}_{\ell,2}(\omega_\ell) h_\ell(x^*_{\ell,2}(\omega_\ell))}{(1 - q_\ell(x^*_{\ell,2}(\omega_\ell)))^2} \tag{75}$$

where $x^{*'}_{\ell,2}(\omega_\ell) = -1/(\omega_\ell^2 \psi^{(i)} \kappa_\ell^{-1})(g_\ell^{-1})'(\omega_\ell^{-1}\kappa_\ell/\psi^{(i)}) < 0$. Moreover, due to (41), we are only interested in the values of $x^{*'}_{\ell,2}(\omega_\ell)$ such that $h_\ell(\omega_\ell) < 0$. As a consequence, $\mathcal{F}_\ell(\omega_\ell)$ is strictly decreasing and hence $\mathcal{M}_\ell(\omega_\ell)$ is strictly increasing, which concludes the proof.

APPENDIX F
PROOF OF LEMMA 6

Let us define k'_m a one-to-one mapping from $\{1, \ldots, L\}$ in itself such that $t^T_{k'_1} \leq \cdots \leq t^T_{k'_L}$ where $t^T_{k'_i}$ is defined in (44). To prove Theorem 6, we first observe that the first term in the right hand side (RHS) of $\tilde{\Gamma}(t)$ is continuous and strictly increasing on every open set $(t^T_{k'_i}, t^T_{k'_{i+1}})$.

Second, let us prove that the second term is also strictly increasing. To this end, we remind that $\gamma^*_{\ell,2}(t)$ is expressed as

$$\gamma^*_{\ell,2}(t) = \frac{\eta_\ell^{(0)}}{\alpha_\ell(1 - q_\ell(x^*_{\ell,2}(\mathcal{M}_\ell^{-1}(t))))}. \tag{76}$$

Since $\mathcal{M}_\ell^{-1}(t)$ is strictly increasing and $x^*_{\ell,2}(\tilde{\omega}_\ell)$ is strictly decreasing, we infer that $1-q_\ell(x^*_{\ell,2}(\mathcal{M}_\ell^{-1}(t)))$ is decreasing and as a consequence $\gamma^*_{\ell,2}(t)$ is strictly increasing. Third, it can be checked that $\tilde{\Gamma}(t)$ is continuous in every $t^T_{k'_i}$ by checking that $\lim_{t\nearrow t^T_{k'_i}} \tilde{\Gamma}(t) = \lim_{t\searrow t^T_{k'_i}} \tilde{\Gamma}(t)$. Finally, by letting $\tilde{t}$ be sufficiently small, one can show that that $\gamma^*_{\ell,2}(t)$ goes to $\eta_\ell^{(0)}/\alpha_\ell$, and we have $\sum_{\ell=1}^{L} \eta_\ell^{(0)}/\alpha_\ell \leq 1$ (otherwise the problem would be infeasible). Moreover, when t is sufficiently large, it is clear that $\tilde{\Gamma}(t) > 1$. Hence, there exists t^* such that $\tilde{\Gamma}(t^*) = 1$, which concludes the proof.

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