Spatio-Temporal Wireless D2D Network With Beamforming

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***Abstract*—In this paper, we consider a dynamic device-to-device (D2D) communication model where transmitters and receivers have multiple antennas and adopt beamforming (BF). A continuous spatio-temporal model for the wireless network is analyzed, which combines a spatial stochastic point process and a dynamic birth-death process. We model BF by using a uniform linear array (ULA) and extend the result of Sankararaman and Baccelli on the stability condition of such a network. We show that the critical arrival rate increases with the number of antennas at the transmitter and the receiver.**

***Index Terms*—Stochastic geometry, birth-death process, beamforming, stability, device-to-device**

I. INTRODUCTION

Device-to-device (D2D) communications emerge as a promising technology to improve the spectral efficiency of next generation cellular networks. Since it allows direct communication between nearby devices, this technique is the basis of new proximity based services [1]. In in-band mode, D2D devices share the same spectrum resource and interfere each other. As the number of D2D devices increases, it becomes more and more difficult to meet stringent quality of service requirements, such as those foreseen for URLLC in 5G.

A solution to improve the networks performance is beamforming (BF). Several papers in the literature show the great potential of BF for D2D communications for reducing interference and improving network throughput, see e.g. [2] [3]. However, it is still an open issue to study the randomness of large scale networks with D2D communication, when there are massive access of random located users using BF with various communication demands.

Stochastic geometry has recently been widely used to characterize large scale networks with D2D communications [4]–[6]. Devices are supposed to form a Poisson point process (PPP) and performance parameters such as successful transmission probability or network capacity are evaluated. A common limitation of these works is that they focus on the network spatial randomness at a given instant but neglect the temporal randomness. In contrast, references [7]–[9] study the dynamic traffic properties of large scale stochastic cellular networks. Networks are modeled as classical queue interacting problems [10], i.e., the number of users and their locations in the network are assumed to be static all the time. Each user has a buffer to accept the data packets which arrive according to a Poisson process with a random volume. The networks are here modeled as discrete time Markov chains. A limitation of this approach is that there is no random arrival or departure of data sources. Moreover, the full buffer assumption is restrictive as some devices may become idle if they have no data to transmit. A more practical and realistic model states that devices visit the network at random time instants with a random amount of data to be transmitted. This assumption is reasonable, because a user who has finished its file transmission may move to another location at another time instant. Then, it starts a new communication there. At the same time, a new user may arrive in the network and start the communication at a random location. This network can be modeled by a Spatial Birth-Death Process [11], [12]. To the best of our knowledge, [13] is the first work to have modeled a D2D network as a spatial birth-death process. The stability condition of this model is studied by the authors. This model has been recently extended to model the uplink of cellular networks in [14].

In this paper, we propose an extension of the work of [13] to D2D networks with BF. We study its impacts on the performance and in particular on the critical arrival rate of D2D users. Our contributions are thus the following:

- We propose a BF model for D2D communications based on the Uniform Linear Array (ULA) model, which is tractable from a stochastic geometry point of view.
- We derive a new stability condition for the spatial birth-death D2D process as a function of the number of transmit and receive antennas.
- Our numerical results show that BF extends the stability region of D2D networks and that the critical arrival rate is increasing with the number of antennas.

The rest of the paper is organized as follow. In Section II, we introduce the system model for the D2D network with BF and derive the transmission rate of each user. In Section III, we present the stability criterion of the spatial birth-death process with BF. Our numerical results are shown in Section IV. Section V concludes the paper.

II. SYSTEM MODEL

In this section, we fist describe the stochastic model of [13] and then extend it to take into account BF. The model of [13] is referred to the so called ”dipole-model” with a series of transmitter-receiver pairs arriving in the network as a spatial

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Poisson process. Each active transmitter in the space transfers a file to its corresponding receiver. This process models a D2D communication network, where several nearby users use simultaneously the same spectrum resource to communicate. Once a transmitter-receiver pair finishes its file transmission, it becomes idle and disappears from the network.

A. Spatial Birth-Death Process

The area in which D2D users live is a two dimensional Euclidean square plane $\mathbf{S} = [-Q, Q] \times [-Q, Q]$, $\mathbf{S} \subset \mathbb{R}^2$, where the side length of the plane is $2Q$, $Q \in \mathbb{R}^+$. The area of $\mathbf{S}$ is denoted as $|\mathbf{S}|$.

The arrival time instants of the transmitter-receiver pairs are modeled as a stationary Poisson process with arrival rate $\lambda|\mathbf{S}|$, where $\lambda \in \mathbb{R}^+$ is the arrival rate per unit of surface. The positions of the receivers in each pair are i.i.d. uniformly distributed in the plane $\mathbf{S}$. For all the pairs, the length between the transmitter and their corresponding receivers are set to be the same constant T in our studies. Then the transmitter devices are uniformly distributed on the circles centered at the receivers of radius T.

At each time instant t, the positions of the receivers in the network are modeled as a spatial point process $\Phi_t^{Rx} = \{x_1, x_2, ..., x_{N_t}\}$ defined on $(\mathbb{R}^2, \mathcal{B}^2)$, where $\mathcal{B}^2$ is the $\sigma-$algebra on $\mathbb{R}^2$, N_t is the number of pairs and x_i denotes the position of the i-th active receiver. Then the positions of pairs can be interpreted as a marked point process $\Phi_t = \{(x_1, y_1), (x_2, y_2)...(x_{N_t}, y_{N_t})\}$, where the marks y_i denote the transmitters locations of the i-th pair. Assume that this process is simple, i.e., there is no multiple users at the same location. Similarly, we denote the process $\Phi_t^{Tx} = \{y_1, y_2, ..., y_{N_t}\}$ as the point process with respect to the transmitters locations. Given a Borel set A, let $\Phi_t(A)$ denote the number of receivers in A at time t. Hence the number of active pairs in the plane $\{\Phi_t(\mathbf{S})\}$ is also a counting process indexed by the time t.

Once the i-th pair arrives at time b_i, it becomes active and has to transfer a file. The file size L_i in bits is a random variable following an exponential law of mean L bits. The transmission rate of each pair is dynamic following the Shannon rate. At the end of the file transmission, the transmitter-receiver pairs leave the plane. The time at which the i-th pair leaves the plane is denoted as d_i. As a consequence, the sojourn time of pair i is $W_i = d_i - b_i$.

TABLE I
ANTENNA GAIN OF THE APPROXIMATED BF ULA MODEL

Antenna Type[a]	Antenna gain	
	G_{max}	G_{min}
Tx	$2n$	ρ
Rx	$2n^2$	$n * \rho$

[a] Tx and Rx denote the transmitter and the receiver.

B. Beamforming

We adopt a two dimensional approximated ULA beamforming (ULA BF) model shown in Fig. 1 for the transmitters as well as the receivers. The model is made of a main beam of gain G_{max} and a side beam of gain G_{min}. We denote ω the half power beamwidth (HPBW) of the array, which corresponds in the model to the angular aperture of the main beam. For each device pair, boreside directions of the antenna arrays are supposed to be aligned.

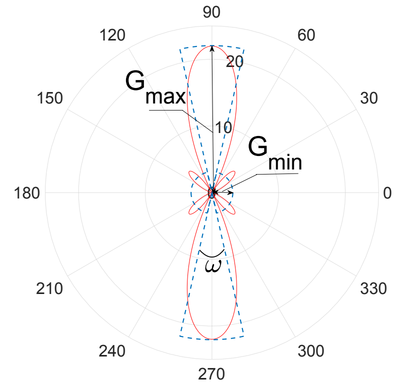

Fig. 1. Uniform Linear Array beamforming (ULA BF) model (solid line) and approximated ULA BF model (dotted line).

The parameters of the approximated ULA BF model are derived from the classical ULA model [15]. Let G_{max}^{Tx} and G_{min}^{Tx} represent the antenna gains of the transmitters antennas. Respectively, we use G_{max}^{Rx} and G_{min}^{Rx} to denote the antenna gain of the receivers antennas. We denote n the number of antenna elements at both transmitters and receivers. The specific relations between G_{max}, G_{min} and n are given in Tab. I, where ρ is the transmitter antenna minimum gain. Both ω and ρ are functions of n, which are shown in (1) and (2). We denote by $K(n)$ the sum of the power gain in the array radiation area. The derivation of (1), (2) and (3) are presented in Appendix A.

$$\omega(n) = 2\left(\frac{\pi}{2} - \arccos\frac{2.784}{n\pi}\right) \approx \frac{1.7723}{n} \quad (1)$$

$$\rho(n) = \frac{K(n) - 2n\omega(n)}{\pi - \omega(n)} \quad (2)$$

$$K(n) = \int_0^{\pi} \frac{2}{n}\left|\frac{\sin(\frac{1}{2}n\pi\cos\theta)}{\sin(\frac{1}{2}\pi\cos\theta)}\right|^2 d\theta \quad (3)$$

Consider two pairs (x_i, y_i) and (x_j, y_j). Let θ_{ij}^{Rx} denote the minimum angle between the direction $\overrightarrow{x_i y_j}$ and pair i's two boreside directions which are denoted by $\overrightarrow{x_i y_i}$ and $\overrightarrow{y_i x_i}$. Respectively the angle θ_{ij}^{Tx} denotes the minimum angle between the direction $\overrightarrow{y_j x_i}$ and the pair j's two boreside directions which are denoted by $\overrightarrow{y_j x_j}$ and $\overrightarrow{x_j y_j}$. Mathematically, they can be expressed as follows:

$$\begin{cases} \theta_{ij}^{Rx} = min(\angle y_j x_i y_i, \pi - \angle y_j x_i y_i) \\ \theta_{ij}^{Tx} = min(\angle x_i y_j x_j, \pi - \angle x_i y_j x_j) \end{cases} \quad (4)$$

Finally, we can define the total power gain from the transmitter y_j to the receiver x_i with BF as G_{ij}.

$$G_{ij} = \begin{cases} G^{Rx}_{max} G^{Tx}_{max} & \textit{if } \theta^{Rx}_{ij} \leq \omega \textit{ and } \theta^{Tx}_{ij} \leq \omega \\ G^{Rx}_{max} G^{Tx}_{min} & \textit{if } \theta^{Rx}_{ij} \leq \omega \textit{ and } \theta^{Tx}_{ij} > \omega \\ G^{Rx}_{min} G^{Tx}_{max} & \textit{if } \theta^{Rx}_{ij} > \omega \textit{ and } \theta^{Tx}_{ij} \leq \omega \\ G^{Rx}_{min} G^{Tx}_{min} & \textit{if } \theta^{Rx}_{ij} > \omega \textit{ and } \theta^{Tx}_{ij} > \omega \end{cases} \quad (5)$$

Since the boreside directions of the transmitter and the receiver of a pair are aligned, we have: $G_{ii} = G^{Rx}_{max} G^{Tx}_{max}$.

C. Transmission Rate

We first consider the case without BF. Let $\ell(r)$ denote the path-loss function at distance r, which is assumed to be bounded. Let P denote the transmission power of the transmitter. Without considering the channel small-scale fading, the power received by the receiver located at x from its corresponding transmitter located at y, can be expressed as $P\ell(\|x - y\|)$. The interference $I(x, \Phi_t)$ is the sum of powers received by a receiver at x from other pairs in the configuration Φ_t:

$$I(x, \Phi_t) = \sum_{u \in \phi^{Tx} \setminus y} P\ell(\|u - x\|) \quad (6)$$

The transmission rate $R(x, \Phi_t)$ of the pair $(x, y) \in \Phi_t$ varies as a function of time according to the Shannon rate:

$$R(x, \Phi_t) = B \log_2 \left(1 + P \frac{\ell(\|x - y\|)}{\mathcal{N}_0 + I(x, \Phi_t)} \right) \quad (7)$$

where B denotes the bandwidth of the transmission channel and $\mathcal{N}_0$ is the thermal noise at the receiver side.

With the application of BF, the received power from transmitter at y_j to receiver at x_i is now $G_{ij} P\ell(\|x_i - y_j\|)$. Hence we get the expression of the interference for a receiver x_i associated with y_i with BF:

$$I^{BF}(x_i, \Phi_t) = \sum_{y_j \in \phi^{Tx}, i \neq j} G_{ij} P\ell(\|y_j - x_i\|) \quad (8)$$

We can thus define the Shannon rate of the transmitter-receiver pair (x_i, y_i) at configuration Φ_t as:

$$R^{BF}(x_i, \Phi_t) = B \log_2 \left(1 + P \frac{G^{Rx}_{max} G^{Tx}_{max} \ell(\|x_i - y_i\|)}{\mathcal{N}_0 + I^{BF}(x_i, \Phi_t)} \right) \quad (9)$$

III. Stability Criterion

In such a spatio-temporal network, all the device pairs are served without constraints of the number of servers. So we can regard this system as a M/G/∞ queue, where the service rate for each user are different and time-varying. The stability of this queue is a critical issue to study. The system is stable if $\Phi_t(\mathbf{S})$ is a stationary counting process. Moreover, Φ_t can be characterized as a continuous-time Markov chain (CTMC) with state space the set of configurations. In brief, the ergodicity, the stationary regime and the stability describe the same phenomenon for such a process. Reference [13] proposes a necessary and sufficient condition to make Φ_t ergodic. We derive the stability criterion (i.e., the time ergodicity) for this process when the approximated ULA BF model presented in Section II-B is adopted at D2D pairs. We use the same definition of stability as in [14].

Definition 1 (Stability)**.** The spatial birth-death process Φ_t is said to be stable, if the number of pairs $\Phi_t(\mathbf{S})$ converges weakly to a limit.

In another word, Φ_t is stable if the embedded Markov chain is time-ergodic [13].

Definition 2 (Critical arrival rate)**.** The critical arrival rate λ_c is defined as the threshold of arrival rate such that, the spatial birth-death process Φ_t is stable if and only if $\lambda < \lambda_c$.

Let $a = \int_{\mathbf{S}} \ell(\|x\|)dx$, the critical arrival rate λ_c given in [13] is expressed as:

$$\lambda_c = \frac{B\ell(T)}{\ln(2)La} \quad (10)$$

Based on this result, we redefine the critical arrival rate under the BF paradigm expressed as follows:

$$\lambda^{BF}_c(n) = \frac{4n^3 B\ell(T)}{\ln(2)La\mathbb{E}[G(n)]} \quad (11)$$

Let $p = \frac{\omega}{\pi}$, then $\mathbb{E}[G]$ is the expectation of a random variable which depends on the number of antennas elements n, ρ and p as follows.

$$\mathbb{E}[G(n)] = 4n^3 p^2 + 4n^2 \rho p(1 - p) + n\rho^2(1 - p)^2 \quad (12)$$

Theorem 1. Assuming the approximated ULA BF model with n antenna elements, if $\lambda > \lambda^{BF}_c(n)$, the spatial birth-death process Φ_t admits no stationary regime.

Proof: See Appendix B.

IV. Simulation

We simulate the birth-death process Φ_t in forward time, based on the system model described in Section II-A. In order to avoid border effects, the network area is transformed in a torus. If Φ_t is stable then it admits a stationary regime Φ_0. We denote β the intensity of this spatial point process Φ_0. The value of β can be estimated by observing the value $\mathbb{E}[\frac{\Phi^0(\mathbf{S})}{\|S\|}]$. We can thus observe the sample path of the intensity function $\beta(t) = \frac{\Phi_t(\mathbf{S})}{\|\mathbf{S}\|}$, which is a time series. If the time average value of $\beta(t)$ converges to be a constant, the process Φ_t is ergodic and has a unique stationary regime. This is known to be the first order property of the stationary point process [14].

The form of the path-loss function $\ell(\cdot)$ should be carefully chosen to keep the value of $a = \int_{x \in \mathbf{S}} \ell(\|x\|)dx$ bounded. In our simulation we take a bounded path-loss model $\ell(r) = (1 + r)^{-4}$. The network plane $\mathbf{S}$ is considered to be a square centered at the origin. The length of $\mathbf{S}$ is $2Q$ and $Q = 10$ meters in our simulation. The pair distance T is chosen as 1 meter. The average file size is $L = 1$ Mbits. Other parameters are the bandwidth $B = 1$ MHz, the noise power $\mathcal{N} = -100$ dBm, and the transmission power $P = 20$ dBm. According to (10), the critical arrival rate without BF is $\lambda_c = 0.0919$ with the above setting.

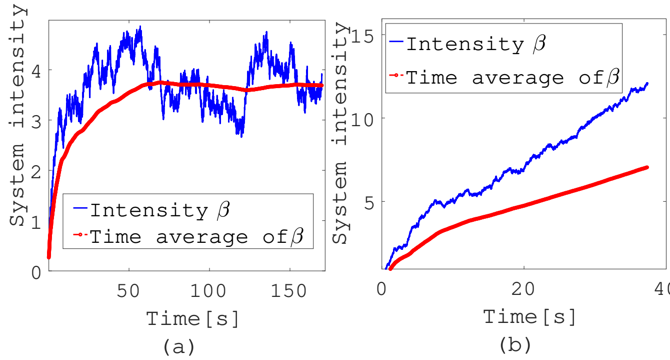

Fig. 2. β as a function of time when $\lambda = 32\lambda_c$ (a), and when $\lambda = 38\lambda_c$ (b), where $n = 4$, $\lambda_c^{BF} = 35.35\lambda_c$.

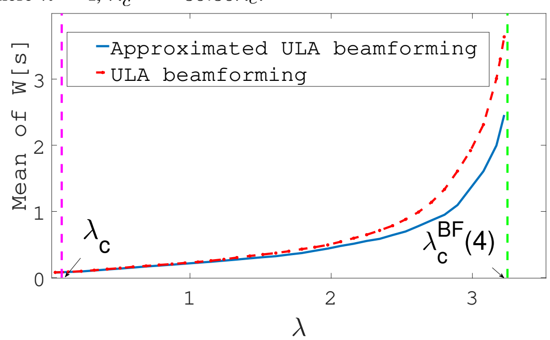

Fig. 3. Average of sojourn time W as a function of λ with $n = 4$. The classical ULA model and the approximated ULA BF model are compared.

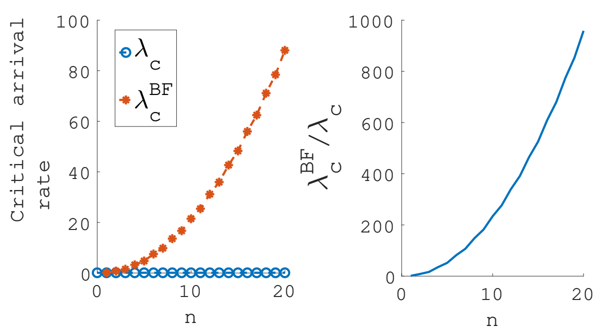

Fig. 4. Critical arrival rates $\lambda_c^{BF}(n)$, λ_c and $\lambda_c^{BF}(n)/\lambda_c$ as a function of n.

The simulation starts with a receiver arriving at a random location in $\mathbf{S}$. Each pair is associated with a file whose size follows the same exponential distribution. We record all the time instants when there are new arrivals or departures, and each pair alive in the network updates its transmission rate at these time instants according to equation (9). We count the number of active pairs $\Phi_t(\mathbf{S})$ in the network at these time instants, and we get $\beta(t) = \frac{\Phi_t(\mathbf{S})}{|\mathbf{S}|}$.

In Fig. 2, we show a case where the number of antenna elements is $n = 4$. According to (11), we get $\lambda_c^{BF} \approx 35.35\lambda_c$. In Fig. 2 (a), we set $\lambda = 32\lambda_c$. The blue curve shows the intensity function $\beta(t)$, and the red curve is its time average trajectory expressed as $\bar{\beta}(t) = \frac{\int_0^t \beta(t)}{t}$. It illustrates clearly that Φ_t admits a stationary regime, since $\bar{\beta}(t)$ reaches a limit. When the arrival rate exceeds $\lambda_c^{BF}(n)$, like in Fig. 2 (b), where $\lambda = 38\lambda_c$, the Markov chain is not time ergodic and $\beta(t)$ grows indefinitely. This is due to the fact that the increase of arrivals causes more interference in the network which reduces the total network throughput, to such extent that the load exceeds the network capacity.

Another approach to verify the stability of the process is to study the sojourn time W. According to the little law, $\beta = W\lambda$ when this system queue is stable. This means that W should be bounded to keep β bounded. The average of W taken among all the pairs in one realisation is shown in Fig. 3. The average of W goes to be infinity when the arrival rate gets close to $\lambda_c^{BF}(n)$, which verifies our stability condition. The figure shows that the average sojourn time using the approximated ULA BF model is a bit smaller than the average sojourn time given by the classical ULA model (14). The approximation gives a good approximation of the average sojourn time and of the critical arrival rate.

The impact of BF is significant in terms of the expansion of the stability region. In Fig. 4, we show $\lambda_c^{BF}(n)$ and λ_c as functions of the number of antenna elements n, where λ_c is a constant. The value of $\lambda_c^{BF}(n)$ increases dramatically when n increases, since (11) is an increasing function of n.

V. Conclusion

In this paper, we present an extension of the wireless network model proposed by Sankararaman and Baccelli based on a spatial birth-death process to the scenario where devices use beamforming. We propose a simple and tractable model for BF based on a uniform linear array and we provide an explicit expression of the critical arrival rate as a function of the number of antenna elements. Theoretical and numerical results show that BF expands significantly the stability region of such a network and that the critical arrival rate is an increasing function of the number of antenna elements.

Appendix A

In this section, we express the two parameters ρ, ω as functions of the number of antenna elements n. The methods is based on [15]. A uniform linear array has n antenna elements aligned along the x-axis and are equally spaced with distance d. Consider a planar wave departing in (or arriving from) the direction θ with respect to the x-axis. The field pattern at every antenna element is denoted by $f_e(\theta)$, the power pattern is denoted by $g_e(\theta)$. Take the phase at the first antenna element as a reference. Considering the antennas at the transmitters part firstly, the combined far field pattern f_a of the array is obtained as:

$$f_a(\theta) \quad = \quad \frac{1}{\sqrt{n}} f_e(\theta) \sum_{i=1}^{n} a_i e^{j((i-1)kd\cos\theta)} \qquad (13)$$

where $k = 2\pi/\lambda$ and a_i is a phase offset applied at antenna element i. The $1/\sqrt{n}$ factor is to account for the power split among the n antenna elements at the transmission. Choosing

$a_i = e^{-j((i-1)kd\cos\theta_0)}$, the power gain $g_a(\theta) = |f_a(\theta)|^2$ leads to:

$$g_a(\theta) = \frac{1}{n} g_e(\theta) \left| \frac{\sin(nk\frac{d}{2}(\cos\theta - \cos\theta_0))}{\sin(k\frac{d}{2}(\cos\theta - \cos\theta_0))} \right|^2 \quad (14)$$

The maximum array factor gain for the transmitter is achieved for $\theta = \theta_0$: the maximum array factor is n. If we assume a rectangular patch antenna, the antenna radiation pattern is hemispheric and has a directivity of 2 [15]. Hence we now assume $\theta_0 = \pi/2$ (boreside), $g_e(\theta) = 2$. Finally the maximum gain of the transmitters antenna is $g_a(\theta_0) = 2n$.

The half power beam width of the array factor is obtained by solving the equation:

$$\frac{2}{n} \left| \frac{\sin(\frac{1}{2}n\pi\cos\theta)}{\sin(\frac{1}{2}\pi\cos\theta)} \right|^2 = \frac{2n}{2} \quad (15)$$

Using an approximation for the sine function for small arguments by approximating the denominator by $(\frac{1}{2}n\pi\cos\theta)^2$, we have to solve: $|\sin(x)/x|^2 = 1/2$. Numerically, we find: $x \approx 1.392$. So that the half power bandwidth (HPBW) $\omega(n)$ of n transmitter antennas is:

$$\omega(n) = 2\left(\frac{\pi}{2} - \arccos\frac{2.784}{n\pi}\right) \approx \frac{1.7723}{n} \quad (16)$$

Let ρ be the minimum gain in the complementary sector. We equalize the approximate and exact radiated powers.

$$2n\omega(n) + \rho(n)(\pi - \omega(n)) = K(n) \quad (17)$$

Then the transmitter antennas minimum gain is given by:

$$\rho(n) = \frac{K(n) - 2n\omega(n)}{\pi - \omega(n)} \quad (18)$$

where $K(n)$ is the total power gain of antennas half radiation region from $\theta = 0$ to $\theta = \pi$ shown by (3).

For the receiver antenna, there is no need to divide the power into n parts. So the received signal is formed as follows:

$$f_a^{Rx}(\theta) = f_e(\theta) \sum_{i=1}^{n} a_i e^{j((i-1)kd\cos\theta)} \quad (19)$$

Using the same method as for the transmitter antenna case, we get that ω has the same value as that we get for transmitters antennas. The minimum gain of receiver's antenna has the value $G_{min}^{Rx} = n\rho$.

Appendix B

In this section, we give the derivation of the critical arrival rate with BF and adapt the proofs given in [13] to our system model. According to the Miyazawa's rate conservative law (RCL) [16], the average rates of increase should be equal to the rates of decrease for a stationary stochastic process. We can thus apply RCL to the number of active pairs in the network:

$$\lambda|\boldsymbol{S}| = \lambda_d. \quad (20)$$

Next we can apply the RCL to the total workload in the network. Denote Φ_0 as the configuration when the network is stationary. Given that the data volume of each pair is i.i.d. exponentially distributed with mean L bits, the RCL leads to the result that:

$$\lambda|\boldsymbol{S}|L = \mathbb{E}\left[\textstyle\sum_{x\in\Phi_0} R^{BF}(x,\Phi_0)\right] \quad (21)$$

$$= \mathbb{E}_G\left[\mathbb{E}\left[\textstyle\sum_{x\in\Phi_0} R^{BF}(x,\Phi_0)\right]|G\right] \quad (22)$$

Where G is the power gain of the transmission path, of which the receiver is located at the origin, and the transmitter is located at a random place. Let $\mathbb{E}_{\Phi_0}^0$ describe the spatial Palm Probability of Φ_0. Since Φ_0 is in steady state this Palm probability must exist [17]. According to the definition of Palm probability, we get:

$$\mathbb{E}\left[\sum_{x\in\Phi_0} R^{BF}(x,\Phi_0)\right] = \mathbb{E}_{\Phi_0}^0[R^{BF}(0,\phi_0)]\mathbb{E}[\phi_0(\boldsymbol{S})] \quad (23)$$

Denote the total interference seen by the receivers in the network as $\mathbf{I}_t^{BF} = \sum_{x\in\Phi_t} I^{BF}(x,\Phi_t)$, it also respects the RCL:

$$\lambda|\mathbf{S}|\mathbb{E}^{\uparrow}[\mathcal{I}] = \lambda_d\mathbb{E}^{\downarrow}[\mathcal{D}] \quad (24)$$

where $\mathcal{I} = \mathbf{I}_{0+} - \mathbf{I}_0$ denotes the additional interference arisen by an arrival. Denote the corresponding palm probability as $\mathbb{E}^{\uparrow}$. Similarly, $\mathcal{D} = \mathbf{I}_0 - \mathbf{I}_{0+}$ denotes the decrease of the interference arisen by a departure and $\mathbb{E}^{\downarrow}$ is its palm probability. Suppose that the system is stationary at Φ_0, the intensity of Φ_0 is $\frac{\mathbb{E}[\Phi_0(\mathbf{S})]}{|\mathbf{S}|}$ if it exists. According to the PASTA property [17], the palm probability can be replaced by the stationary probability when the process is Poisson. Since the births are Poisson and uniform in $\mathbf{S}$, we can suppose that the birth occurs at the origin center of the space. By applying Campbell's theory this leads that:

$$\mathbb{E}^{\uparrow}[\mathcal{I}^{BF}] = 2\mathbb{E}_G[\mathbb{E}[\sum_{x\in\Phi_0} GP\ell(\|x\|)]|G] \quad (25)$$

$$= 2P\frac{\mathbb{E}[\Phi_0(\mathbf{S})]}{|\mathbf{S}|}\mathbb{E}_G[\int_{\mathbf{S}} G\ell(\|x\|)dx] \quad (26)$$

Let $a^{BF} = \int_{\mathbf{S}} G\ell(\|x\|)$ being a random variable which is a function of G. The coefficient 2 in formula (25) comes from the fact that, once a device pair arrives, the total interference it receives from other devices has the same amount as it leads to the network. We get (26) by applying Campbell's formula.

According to (20) and (21), we get the intensity corresponding to the death epochs as follows:

$$\lambda_d = \frac{\mathbb{E}\left[\sum_{x\in\Phi_0} R(x,\Phi_0)\right]}{L} \quad (27)$$

considering the simple point process on the real line corresponding to the death-instants. Different from the birth process, the rate of the death process depends on the configuration of the network which is dynamic. Since the file-size is a stochastic variable with mean L, the point process admits a stochastic-intensity $\lambda_d(t) = \frac{1}{L}\sum_{x\in\Phi_t} R(x,\Phi_t)$ with respect to the filtration $\mathcal{F}_t = \sigma(\phi_s : s \le t)$. It then follows the Papangelou's theorem [17] that

$$\lambda_d\mathbb{E}^{\downarrow}[\mathcal{D}] = \mathbb{E}[\lambda_d(0)\mathcal{D}] \quad (28)$$

The decrease magnitude of total interference also differs from the uniform hypothesis of the birth case. At time 0, the death probability of each pair is different. Denote the event X that a pair leaves $\mathbf{S}$ at time 0^- as F. At static regime Φ_0, the decrease $\mathcal{D}$ has magnitude $I(X, \Phi_0)$ with a probability $\mathbb{P}[X \in F]$. Since the file size are i.i.d. and exponentially distributed with mean L, this probability can be expressed as follows:

$$\mathbb{P}[X \in F] = \lim_{\epsilon \to 0} \frac{1 - \exp\left(-\epsilon R(X, \Phi_0)/L\right)}{1 - \exp\left(-\sum_{x\in\Phi_0} \epsilon R(x, \Phi_0)/L\right)} \quad (29)$$

$$= \frac{R(X, \Phi_0)}{\sum_{x\in\Phi_0} R(x, \Phi_0)} \quad (30)$$

Let $\mathbf{R}_0 = \sum_{x\in\Phi_0} R(x, \Phi_0)$. We can then get the equation below by applying (27), (28) and (30).

$$\mathbb{E}^{\downarrow}[\mathcal{D}] = 2\mathbb{E}\left[\frac{\lambda_d(0)}{\mathbb{E}[\lambda_d(0)]} \sum_{x\in\Phi_0} \frac{R^{BF}(x, \Phi_0)}{\boldsymbol{R}_0^{BF}} I^{BF}(x, \Phi_0)\right] \quad (31)$$

$$= 2\frac{\mathbb{E}[\sum_{x\in\Phi_0} R^{BF}(x, \Phi_0) I^{BF}(x, \Phi_0)]}{\mathbb{E}[\boldsymbol{R}_0^{BF}]} \quad (32)$$

$$= 2\frac{\mathbb{E}^0_{\Phi_0}[R^{BF}(0, \Phi_0) I^{BF}(0, \Phi_0)]}{\mathbb{E}[\boldsymbol{R}_0^{BF}]} \mathbb{E}[\Phi_0(\mathbf{S})] \quad (33)$$

Now combine (20) and (24), we get $\mathbb{E}^{\downarrow}[\mathcal{D}] = \mathbb{E}^{\uparrow}[\mathcal{I}]$. Thus we get:

$$\begin{aligned} &2\frac{\mathbb{E}[\Phi_0(\mathbf{S})]}{|\mathbf{S}|}\mathbb{E}_G[a^{BF}] \\ =&2\frac{\mathbb{E}^0_{\Phi_0}[R^{BF}(0, \Phi_0) I^{BF}(0, \Phi_0)]}{\mathbb{E}[\boldsymbol{R}_0^{BF}]} \mathbb{E}[\Phi_0(\mathbf{S})] \end{aligned} \quad (34)$$

By applying (21) to (34), we obtain:

$$\mathbb{E}_G[a^{BF}] = \frac{\mathbb{E}^0_{\Phi_0}[R^{BF}(0, \Phi_0) I^{BF}(0, \Phi_0)]}{L\lambda} \quad (35)$$

Given the definition of the transmission rate in (9), we have:

$$R^{BF}(0, \Phi_0) I^{BF}(0, \Phi_0) \leq \frac{BG^{Rx}_{max} G^{Tx}_{max} \ell(T)}{ln(2)} \quad (36)$$

Applying this inequality to (35), we get the following inequality:

$$\lambda \leq \frac{BG^{Rx}_{max} G^{Tx}_{max} \ell(T)}{\ln(2) L \mathbb{E}_G[a^{BF}]}. \quad (37)$$

Now we define the probability distribution of the transmission gain G. Based on formula (4), let p denote the probability that $\theta^{Rx}_{ox} \leq \omega$ or $\theta^{Tx}_{ox} \leq \omega$. Because the transmitters are uniformly located around the receiver, and the boreside of antennas are aligned, we get $p = \frac{\omega}{\pi}$. Then we can get the expectation of $G(n)$ based on the formula (5) and the value of G_{max}, G_{min} in Tab. I. The expression of $\mathbb{E}[G(n)]$ is given in (12). Thus a^{BF} has the expectation as below:

$$\mathbb{E}_G[a^{BF}] = \mathbb{E}[G(n)]a \quad (38)$$

According to the relation between the number of antenna elements and the gain, we conclude that the critical arrival rate is as follows:

$$\lambda^{BF}_C(n) = \frac{BG^{Rx}_{max} G^{Tx}_{max} \ell(T)}{\ln(2) L \mathbb{E}[G(n)] a} \quad (39)$$

$$= \frac{4n^3 B \ell(T)}{\ln(2) La \mathbb{E}[G(n)]} \quad (40)$$

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